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THESE RESULTS ARE THE ANSWERS TO THE PROBLEM PROPOSED BY E.M.LANDIS FOR 2-ORDERED POLYHARMONIC FUNCTIONS

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ABSTRACT

In this work, the polygarmonious functions of the n th order satisfying the condition given in some unlimited set of m-dimensional space are considered, having received an integral representation with the help of it, fragmen-Lindelof theorems are obtained and the solution of the regularization problem is considered $2n \geq m$.

Key words: *biharmonic functions, polygarmonious functions, Carleman's function, unbounded domain, the regularization problem, integral representation.*

НЕКОТОРЫЕ РЕЗУЛЬТАТЫ КОТОРЫЕ ЯВЛЯЮТСЯ ОТВЕТОМ НА ЗАДАЧУ, ПРЕДЛОЖЕННУЮ Э.М.ЛАНДИСОМ ДЛЯ ПОЛИГАРМОНИЧЕСКИХ ФУНКЦИЙ 2-ГО ПОРЯДКА

АННОТАЦИЯ

В этой статье мы рассматриваем функции полигармонических функций n ного порядка определенных в неограниченной m-мерной области евклидова пространство, с получением интегрального представления получим теоремы типа Фрагмена-Линделефа и регуляризационние решение задачи при 2n≥m.

Ключевые слова: *Гармонические, бигармоническые функции, функция Карлемана, полигармонические функции, регуляризационние решение, частные производные, нормальные производные.*

E.M.LANDIS TOMONIDAN 2 TARTIBLI POLIHARMONIK FUNKSIYALAR UCHUN BERILINGAN MASALAGANING

ANNOTATSIYA

Ushbu ishda m-o‘lchovli fazoning ba’zi cheksiz to‘plamida berilgan shartni qanoatlantiruvchi n-tartibli ko‘p garmoniyali funktsiyalar ko‘rib chiqilib, uning yordamida integral ko‘rinish olingan, fragmen-Lindelof teoremalari olingan va yechimi. tartibga solish muammosi 2n≥m deb hisoblanadi.

Kalit so‘zlar: *bigarmonik funksiyalar, poligarmonli funksiyalar, Karleman funksiyasi, cheksiz soha, tartibga solish muammosi, integral tasvir.*

We shall result in this clause the theorem for some polyharmonic functions determined in an unlimited domain.

In the given work is discussed continuations polyharmonic of function $u(x)$, on its meanings, and meanings of its normal border S, derivative on a smooth part, of infinite domain D.

Let R^m - material space, $x = (x_1, x_2, x_3, \dots, x_m)$, $x \in R^m$, $y \in R^m$, $x' = (x_1, x_2, \dots, x_{m-1}, 0)$, $r = |x - y|$, $s = |x' - y'|$, $h = \pi/\rho$, $\rho > 0$, $\alpha^2 = s$,

D -the unlimited area lying in a layer $\{y: y = (y_1, y_2, \dots, y_m), y_j \in R, j = 1, \dots, m - 1, 0 < y_m < h\}$ with border, $\partial D = \{y: y = (y_1, y_2, \dots, y_m), y_m = 0\} \cup S, S = \{y: y = (y_1, \dots, y_m),\ y_m = f(y_1, \dots, y_{m-1})\}$ where $f(y_1, \dots, y_{m-1})$ has limited private derivative of the first order.

Problem Cauchy. Let $u \in C^{2n}(D)$ and $\Delta^n u(y) = 0, y \in D$ (1)

$$u(y) = F_0(y), \quad \Delta u(y) = F_1(y), \dots, \Delta^{n-1} u(y) = F_{n-1}(y), \quad y \in S$$

$$\frac{\partial u(y)}{\partial \bar{n}} = G_0(y), \quad \frac{\partial \Delta u(y)}{\partial \bar{n}} = G_1(y), \ldots, \frac{\partial \Delta^{n-1} u(y)}{\partial \bar{n}} = G_{n-1}(y), \quad y \in S, \quad (2)$$

Where $F_i(y), G_i(y)$ given on ∂D continuous function, $\bar{n}$-external normal to ∂D. It is required to restore $u(y)$ in D.

For arbitrary initial facts (Cauchy data) this problem is unsolvable. If part of the boundary and initial facts are analytics, then extension to D exist and unique, but not steady. Thus, this problem belongs to incorrect posted problems.

Already in 1943, A.N Tikhonov showed practical importance of unsteady problems and showed, if do narrowing class of all solution to compact, then problem became steady.

Based on this researches M.M Lavrentyev introduce important conception – Carleman function and using it constructed the regularization of problem. Using ideas of M.M Lavrentyev Sh. Yormuxamedov constructed the Carleman function, which can regularize the solution of the problem Cauchy for Laplace equation in bounded domains [2]. In 1992 [6] Ashurova Z.R. proved the theorem:

Let D be an unbounded simply connected domain lying, lies within the layer $0 \le y_m \le \frac{\pi}{\rho}, \rho > 0$ b the border ∂D - surface, any finite part satisfies the Lyapunov condition and, moreover, let outside some ball border ∂D is given by a finite number of single-valued functions having first-order partial derivatives. If $u(y) \in A_\rho(D)$ performed by the condition

$u(y) = 0, \left|\frac{\partial u(y)}{\partial n}\right| < |y|^\mu, \mu > 0, , \forall y \in \partial D$

then $\forall x \in D \;\; u(x) = 0$.

Then in 2009 N. Juraeva got regularization and solved Cauchy task for n-ordered polyharmonic function in some unbounded domains (for arbitrary odd m and even, when - 2n < m).[3]. Later, Juraeva N.Y. with Juraeva U.Y. proved similar results. In this work constructed Carleman function for unbounded domains of m- even-dimensional spaces, when 2n ≥ m and using it taken Phragmen-Lindelof type theorems. [4].

Later, for some areas and for certain classes of functions, similar results were obtained. [5]- [11]. In 2022 Juraeva U.Y. theorems “Phragmen-Lindelof type theorems” [12].

In this paper, we solve similar results for polyharmonic functions

Function $\Phi_\sigma(y,x)$ we can define by the following equality:

$$\Phi_\sigma(y,x) = C_{n,m} \int_{\sqrt{s}}^{\infty} Im \left[\frac{exp(\sigma w + w^2) - achi\rho_1 \left(w - \frac{h}{2}\right)}{\omega - x_1} \right] (u^2 - s)^{n-k} du, \;(3)$$

$\omega = iu + y_1,\ C_{n,m} = (-1)^{\frac{m}{2}-1}\left(\Gamma(n-\frac{m}{2}+1)2^{2n-1}\pi^{\frac{m}{2}}\Gamma(n)\right)^{-1},$

so $\exp\left(\sigma\omega + \omega^2 - a\cos\rho_1\left(\omega - \frac{h}{2}\right)\right) =$

$= \exp\left[\sigma(iu + y_m) + (iu + y_m)^2 - a\cos\rho_1\left(iu + y_m - \frac{h}{2}\right)\right] = \exp\left[\sigma y_m + y_m^2 - u^2 - -ach\rho_1 u\cos\rho_1\left(y_m - \frac{h}{2}\right) + i\left(\sigma u + 2uy_m + ash\rho_1 u\sin\rho_1\left(y_m - \frac{h}{2}\right)\right)\right] =$

$= \exp\left[\sigma y_m + y_m^2 - u^2 - ach\rho_1 u\cos\rho_1\left(y_m - \frac{h}{2}\right)\right]\exp i\left(\sigma u + 2uy_m + ash\rho_1 u\sin\rho_1\left(y_m - \frac{h}{2}\right)\right)$

$= \exp\left|\sigma y_m + y_m^2 - u^2 - ach\rho_1 u\cos\rho_1\left(y_m - \frac{h}{2}\right)\right|(\cos A_2 + i\sin A_2)$

where

$A_1 = \sigma y_m + y_m^2 - u^2 - ach\rho_1 u\cos\rho_1\left(y_m - \frac{h}{2}\right),$

$$A_2 = \sigma u + 2uy_m - ash\rho_1 u\sin\rho_1\left(y_m - \frac{h}{2}\right)$$

According to this

$$u^2 - s = r^2t^2 - r^2, u^2 = r^2t^2 - (r^2 - s) = r^2t^2 - (y_m - x_m)^2,$$

$$u = \sqrt{r^2t^2 - (y_m - x_m)^2}, \qquad 2udu = 2r^2tdt, du = \frac{r^2tdt}{\sqrt{r^2t^2 - (y_m - x_m)^2}}$$

$$A_1 = \sigma y_m + y_m^2 - st^2 - ach\rho_1 t\sqrt{s}\cos\rho_1\left(y_m - \frac{h}{2}\right)$$

$A_2 = \sigma t\sqrt{s} + 2t\sqrt{s}y_m - ash\rho_1 t\sqrt{s}\sin\rho_1\left(y_m - \frac{h}{2}\right),$

it has been taken:

Theorem–1. Function $\Phi_\sigma(y,x)$ can be defined by the following equality (3) - polyharmonic of functions. (s > 0).

Proof:

$$\Phi_\sigma(y,x) = \int_{\sqrt{s}}^{\infty}\left[\frac{exp(\sigma y_m^2)(y_m - x_m)\sin A_2}{exp\left(\sigma u^2 + \left(ach(\rho_1 u) + bch(\rho_0 u)\right)\cos\left(y_m - \frac{h}{2}\right)\right)((y_m - x_m)^2 + u^2)}\right](u^2 - s)^{n-k}du -$$

$$- \int_{\sqrt{s}}^{\infty}\left[\frac{exp(\sigma y_m^2)\, u\cos A_2}{exp\left(\sigma u^2 + \left(ach(\rho_1 u) + bch(\rho_0 u)\right)\cos\left(y_m - \frac{h}{2}\right)\right)((y_m - x_m)^2 + u^2)}\right](u^2 - s)^{n-k}du$$

$$\Phi_\sigma(y,x) = \int_{\sqrt{s}}^{\infty}\left[\frac{exp\left(\sigma y_m^2 - \left(ach(\rho_1 u) + bch(\rho_0 u)\right)cos(y_m - h/2)\right)\left((y_m - x_m)\sin F - u\cos F\right) - 1}{exp(\sigma u^2)\left((y_m - x_m)^2 + u^2\right)}\right](u^2 - s)^{n-k}du +$$

$$+\int_{\sqrt{s}}^{\infty}\frac{1}{exp(\sigma u^2)\left((y_m - x_m)^2 + u^2\right)}(u^2 - s)^{n-k}du$$

$$\Phi_\sigma(y,x) = G_\sigma(y,x) + \int_{\sqrt{s}}^{\infty}\frac{1}{exp(\sigma u^2)\left((y_m - x_m)^2 + u^2\right)}(u^2 - s)^{n-k}du$$

Theorem–2. For function $\Phi_\sigma(y,x)$ takes place (s > 0).

$$|\Phi_\sigma(y,x)| = \left|\alpha^{2(n-k)+1}(J_1 + J_2)\right| \le \frac{c_0\sigma}{exp\left(ach\rho_1\alpha\cos\rho_1\left(y_m - \frac{h}{2}\right)\right)}\left(\frac{1}{\alpha} + \frac{1}{\alpha^2}\right)$$

Proof:

$$u^2 - s = st^2 - s\ ,\quad u^2 = st^2,\qquad u = \alpha t$$

$$\exp\left[\sigma y_m + y_m^2 - u^2 - ach\rho_1 u\cos\rho_1\left(y_m - \frac{h}{2}\right) + i\left(\sigma u + 2uy_m + ash\rho_1 u\sin\rho_1\left(y_m - \frac{h}{2}\right)\right)\right]$$

$$\Phi_\sigma(y,x) = c_0 s^{(n-k)+\frac{1}{2}}\int_1^{\infty}\frac{exp(\sigma y_m + {y_m}^2)\alpha\cos(\sigma\alpha t + 2\alpha t y_m + sh\rho_1\alpha t sin\rho_1\beta_2)(t^2 - 1)^{n-k}tdt}{\left((y_m - x_m)^2 + \alpha^2 t^2\right)exp(st^2 + ach\rho_1\alpha t sin\rho_1\beta_2)} +$$

$$+c_0 s^{(n-k)+\frac{1}{2}}\int_1^{\infty}\frac{exp(\sigma y_m + {y_m}^2)\alpha\sin(\sigma\alpha t + 2\alpha t y_m + sh\rho_1\alpha t sin\rho_1\beta_2)(t^2 - 1)^{n-k}tdt}{\left((y_m - x_m)^2 + \alpha^2 t^2\right)exp(st^2 + ach\rho_1\alpha t sin\rho_1\beta_2)}$$

$$J_2 = exp(\sigma y_m + y_m^2)(y_m - x_m)\int_1^{\infty} \frac{sin(\sigma\alpha t + sh\rho_1\alpha\, t sin\,\rho_1\,\beta_2)(t^2-1)^{n-k}dt}{((y_m - x_m)^2 + \alpha^2 t^2)\, exp\left(st^2 + ach\rho_1\alpha t\, cos\,\rho_1\left(y_m - \frac{h}{2}\right)\right)}$$

$$|cos(\sigma\alpha t + sh\rho_1\alpha\, t sin\,\rho_1\,\beta_2)| \leq$$

$$\leq |cos(\sigma\alpha t) cos\,(sh\rho_1\alpha\, t sin\,\rho_1\,\beta_2)| + |sin(\sigma\alpha t) sin\,(sh\rho_1\alpha t\, sin\,\rho_1\,\beta_2)| \leq c_0\sigma\alpha t$$

$$\int_1^{\infty}\left|\frac{\alpha\, cos(\sigma\alpha t + sh\rho_1\alpha\, sin\,\rho_1\,\beta_2)\quad (t^2-1)^{n-k} t dt}{((y_m - x_m)^2 + \alpha^2 t^2)\, exp\left(st^2 + ach\rho_1\alpha t\, cos\,\rho_1\left(y_m - \frac{h}{2}\right)\right)}\right| \leq$$

$$\leq \frac{c_0\sigma}{exp\left(ach\rho_1\alpha\, cos\,\rho_1\left(y_m - \frac{h}{2}\right)\right)}\int_1^{\infty}\left|\frac{(t^2-1)^{n-k}dt}{exp(st^2)}\right| \leq$$

$$\leq \frac{c_0\sigma}{exp\left(ach\rho_1\alpha\, cos\,\rho_1\left(y_m - \frac{h}{2}\right)\right)}\frac{1}{(\alpha^2)^{n-k+1}}$$

$$|sin(\sigma\alpha + sh\rho_1\alpha\, sin\,\rho_1\,\beta_2)| \leq$$

$$\leq \left(sin(\sigma\alpha t) cos\,(sh\rho_1\alpha\, t sin\,\rho_1\,\beta_2) + sin(sh\rho_1\alpha t\, sin\,\rho_1\,\beta_2) cos\,(\sigma\alpha t)\right) \leq c_0\sigma\alpha t$$

$$\left|\int_1^{\infty}\frac{sin(\sigma\alpha t + sh\rho_1\alpha\, sin\,\rho_1\,\beta_2)(t^2-1)^{n-k}dt}{((y_m - x_m)^2 + \alpha^2 t^2)\, exp\left(st^2 + ach\rho_1\alpha t\, cos\,\rho_1\left(y_m - \frac{h}{2}\right)\right)}\right| \leq$$

$$\left|\int_1^{\infty}\frac{\alpha t}{\alpha^2 t^2}\frac{\sigma(t^2-1)^{n-k}dt}{exp\left(st^2 + ach\rho_1\alpha t\, cos\,\rho_1\left(y_m - \frac{h}{2}\right)\right)}\right| \leq$$

$$\leq \frac{c_0\sigma}{exp\left(ach\rho_1\alpha\, cos\,\rho_1\left(y_m - \frac{h}{2}\right)\right)}\int_1^{\infty}\left|\frac{(t^2-1)^{n-k}dt}{\alpha\;\; exp(st^2)}\right| \leq$$

$$\leq \frac{c_0\sigma}{exp\left(ach\rho_1\alpha\, cos\,\rho_1\left(y_m - \frac{h}{2}\right)\right)}\frac{1}{\alpha^{2(n-k+1)+1}}$$

$$|\Phi_\sigma(y,x)| = \left|\alpha^{2(n-k)+1}(J_1+J_2)\right| \le \frac{c_0\sigma\alpha^{2(n-k)+1}}{exp\left(ach\rho_1\alpha\cos\rho_1\left(y_m-\frac{h}{2}\right)\right)}\left(\frac{1}{(\alpha^2)^{n-k+1}}+\frac{1}{\alpha^{2(n-k+1)+1}}\right) \le$$

$$\le \frac{c_0\sigma}{exp\left(ach\rho_1\alpha\cos\rho_1\left(y_m-\frac{h}{2}\right)\right)}\left(\frac{1}{\alpha}+\frac{1}{\alpha^2}\right).$$

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