YARIM O‘QDA BERILGAN SHTURM-LIUVILL CHEGARAVIY MASALASI HAMDA VEYL-TITCHMARSH FUNKSIYASI VA SPEKTRAL FUNKSIYA ORASIDAGI BOG‘LANISH

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ANNOTATSIYA

Ushbu maqolada yarim o‘qda berilgan SHturm-Liuvill chegaraviy masalasi hamda Veyl-Titchmarsh funksiyasi va spektral funksiya orasidagi bog‘lanish o‘rganiladi.

Kalit so‘zlar: Yarim o‘qda berilgan SHturm – Liuvill chegaraviy masalasi, Veyl-Titchmarsh funksiya, Spektral funksiya, Xos qiymat.

ABSTRACT

This paper examines the Sturm-Liouville boundary value problem given in the semicircle and the relationship between the Weil-Titchmarsh function and the spectral function.

Key words: Sturm-Liouville boundary value problem, eigenvalue, spectral function, spectr, direct problem.

Teorema 1. Agar m(τ) – Veyl nuqtasi yoki veyl doirasiga tegishli bo‘lgan biror nuqta bo‘lsa, u holda ixtiyoriy $\tau \in C\backslash R$ kompleks son uchun ushbu

$$\begin{cases} -y'' + q(x)y = \tau(y), \\ y'(0) = hy(0), \end{cases}$$ (0≤ $x \le \infty$), (bu yerda q(x)∈ $[0, \infty)$ haqiqiy funksiya,

h ixtiyoriy haqiqiy son va τ kompleks parametr) tenglamaning

$$\varphi(x,\tau) = \theta(x,\tau) + m(\tau)\omega(x,\tau)$$

yechimi $L^2(0,\infty)$ fazoga tegishli bo‘ladi hamda quydagi tengsizlikni qanoatlantiradi:

$$\int_0^\infty |\varphi(x,\tau)|^2 dx \le -\frac{Imm(\tau)}{Im\tau}$$

Ta'rif 1. $\varphi(x,\tau) = \theta(x,\tau) + m(\tau)\omega(x,\tau)$ yechimga Veyl yechimi, m(τ) funksiyaga esa Veyl-Titchmarsh funksiyasi deyladi.

Teorema 2. *Agar (a,b) oraliqning chetki nuqtalari $\rho(\lambda)$ spektral funksiyaning uzluksizlik nuqtalaridan iborat bo'lsa, u holda*

$$\rho(b) - \rho(a) = -\frac{1}{\pi}\int_a^b \mathrm{Im}\{m(u+iv\}\,du \quad (2)$$

tenglik o'rinli bo'ladi.

2-Natija. Spektral funksiyaning xos qiymatdagi sakrash uzunligi Veyl-Titchmarsh funksiyasining shu nuqtadagi chegirmasiga teng:

$$\rho(\lambda_0 + 0) - \rho(\lambda_0 - 0) = \operatorname*{res}_{\lambda=\lambda_0} m(\lambda). \quad (2.1)$$

Ushbu

$$-y'' + q(x)y = \lambda y, \qquad (0 \le x \le b), \quad (3.1)$$

$$y(0)\cos\alpha + y'(0)\sin\alpha = 0, \quad (3.2)$$

chegaraviy masalani qaraymiz. Bu yerda $q(x) \in C[0,\infty)$ haqiqiy funksiya, $\alpha \in R^1$ haqiqiy son.

Agar $\sin\alpha \ne 0$ bo'lsa, u holda (3.2) chegaraviy shartni

$$y'(0) - hy(0) = 0, \qquad h = -ctg\,\alpha, \quad (3.3)$$

ko'rinishda yozish mumkin. (3.1) formula (3.2) chegaraviy shart bajarilganda keltirib chiqarilgan edi. Shuning uchun

$m(z) = -\mathrm{ctg}\,\alpha + \int_{-\infty}^{\infty}\frac{d\rho(\lambda)}{z-\lambda}$, $(\sin\alpha \ne 0)$ formuladagi $m(z)$ va $\rho(\lambda)$ funksiyalarni mos ravishda $m_\alpha(z)$ va $\rho_\alpha(\lambda)$ orqali belgilaymiz. Agar (3.1)+(3.3) chegaraviy masalaning spektral funksiyasini $\rho_h(\lambda)$ orqali belgilasak, u holda $\rho_\alpha(\lambda)$ va $\rho_h(\lambda)$ funksiyalar o'zaro quyidagi formulalar bilan bog'langan:

$$\rho_\alpha(\lambda) = \frac{1}{sin^2\alpha} \cdot \rho_h(\lambda), \tag{3.4}$$

Shuning uchun (3.1) formulani quyidagicha ko‘rinishda yozish mumkin:

$$m_\alpha(z) = -ctg\alpha + \int_{-\infty}^{\infty} \frac{d\rho_\alpha(\lambda)}{z-\lambda} = h + \frac{1}{sin^2\alpha} \int_{-\infty}^{\infty} \frac{d\rho_h(\lambda)}{z-\lambda}. \tag{3.5}$$

Ikkinchi tomondan $\rho_h(\lambda)$ spektral funksiya uchun quyidagi asimptotik formula o‘rinli bo‘ladi:

$$\rho_h(\lambda) = \frac{2}{\pi}\sqrt{\lambda} - h + \rho_h(-\infty) + \bar{\bar{o}}(1), \tag{3.6}$$

Spektral funksiyaning bu asimptotik formulasidan va (3.5) formuladan Veyl-Titchmarshning $m_\alpha(z)$ funksiyasi uchun asimptotik formula topishimiz mumkin.

Teorema 3. (3.1) + (3.2) chegaraviy masalaning $m_\alpha(z)$ Veyl-Titchmarsh funksiyasi uchun ushbu $\delta < \arg z < \pi - z$ sohada quyidagi asimptotik formula o‘rinli:

$$m_\alpha(z) = -ctg\alpha - \frac{i}{\sqrt{z}sin^2\alpha} + \frac{ctg\alpha}{zsin^2\alpha} + \bar{\bar{o}}\left(\frac{1}{z}\right), \quad |z| \to \infty, \tag{3.7}$$

Bu yerda $\delta > 0$ - biror musbat son.

Misol: Ushbu

$$\begin{cases} -y'' - \dfrac{2a^2}{ch^2ax} y = \lambda y, & 0 \le x < \infty \\ y'(0) - hy(0) = 0, \end{cases}$$

Shturm-Liuvill masalasining spektral funksiyasini topamiz.

Ma’lumki masalada berilgan differensial tenglamaning umumiy yechimi quyidagi ko‘rinishda ifodalanadi:

$$y = C_1\left(\cos\sqrt{\lambda}x - athax\frac{\sin\sqrt{\lambda}x}{\sqrt{\lambda}}\right) + C_2\left(\sqrt{\lambda}\sin\sqrt{\lambda}x + athax\cos\sqrt{\lambda}x\right).$$

Quyidagi

$$\begin{cases}\theta(x,0)=0\,,\\ \theta'(x,0)=1,\end{cases}\qquad \begin{cases}\varphi(x,0)=1\,,\\ \varphi'(x,0)=h\end{cases}$$

boshlanich shartlarni qanoatlantiruvchi yechimlari quyidagicha ko‘rinishda bo‘ladi:

$$\theta(x,\lambda)=\frac{1}{\lambda+a^2}\left(\sqrt{\lambda}\sin\sqrt{\lambda}x+athax\cos\sqrt{\lambda}x\right),$$

$$\varphi(x,\lambda)=\cos\sqrt{\lambda}x-ath(ax)\frac{\sin\sqrt{\lambda}x}{\sqrt{\lambda}}+\frac{h}{\lambda+a^2}\left(\sqrt{\lambda}\sin\sqrt{\lambda}x+athax\cos\sqrt{\lambda}x\right),$$

$q(x)\equiv 0$ koeffitsiyent quyidan chegaralanganligi uchun Veylning nuqta holi o‘rinli bo‘ladi.

Ushbu

$\psi(x,\lambda)=\theta(x,\lambda)+m(\lambda)\varphi(x,\lambda)$ funksiya $L^2(0,\infty)$ fazoga tegishli bo‘ladigan qilib, $m(\lambda)$ funksiyani tanlaymiz. Buning uchun quyidagicha ifodalashni bajaramiz:

$$\theta(x,\lambda)+m(\lambda)\varphi(x,\lambda)=\frac{1}{\lambda+a^2}\left(\sqrt{\lambda}\sin\sqrt{\lambda}x+athax\cos\sqrt{\lambda}x\right)+$$

$$+m(\lambda)\left(\cos\sqrt{\lambda}x-ath(ax)\frac{\sin\sqrt{\lambda}x}{\sqrt{\lambda}}+\frac{h}{\lambda+a^2}\left(\sqrt{\lambda}\sin\sqrt{\lambda}x+athax\cos\sqrt{\lambda}x\right)\right)=$$

$$=\frac{1}{2}\left\{\frac{athax}{\lambda+a^2}-\frac{i\sqrt{\lambda}}{\lambda+a^2}+m(\lambda)+\frac{i}{\sqrt{\lambda}}m(\lambda)athax-\frac{ih\sqrt{\lambda}}{\lambda+a^2}m(\lambda)\right.$$
$$\left.+\frac{h}{\lambda+a^2}m(\lambda)athax\right\}e^{i\sqrt{\lambda}x}+$$
$$+\frac{1}{2}\left\{\frac{athax}{\lambda+a^2}+\frac{i\sqrt{\lambda}}{\lambda+a^2}+m(\lambda)-\frac{i}{\sqrt{\lambda}}m(\lambda)athax+\frac{ih\sqrt{\lambda}}{\lambda+a^2}m(\lambda)\right.$$
$$\left.+\frac{h}{\lambda+a^2}m(\lambda)athax\right\}e^{-i\sqrt{\lambda}x}, \qquad (4)$$

Kompleks ildizning ushbu

$$\sqrt{u+iv}=\sqrt{\frac{u+\sqrt{u^2+v^2}}{2}}+i\sqrt{\frac{-u+\sqrt{u^2+v^2}}{2}}\,,\quad v>0,$$

shoxchasini tanlash qabul qilinganligi uchun $\operatorname{Im}\lambda>0$ bo‘lganda

$e^{i\sqrt{\lambda}x}\in L^2(0,\infty)$ va $e^{-i\sqrt{\lambda}x}\notin L^2(0,\infty)$, bo‘ladi. Demak, (4) tenglikda ikkinchi qavs ichidagi ifodani nolga tenglaymiz:

$$\frac{athax}{\lambda+a^2}+\frac{i\sqrt{\lambda}}{\lambda+a^2}+m(\lambda)-\frac{i}{\sqrt{\lambda}}m(\lambda)athax+\frac{ih\sqrt{\lambda}}{\lambda+a^2}m(\lambda)+$$
$$+\frac{h}{\lambda+a^2}m(\lambda)athax=0\,,$$

$$\frac{a}{\lambda+a^2}+\frac{i\sqrt{\lambda}}{\lambda+a^2}+m(\lambda)-\frac{ia}{\sqrt{\lambda}}m(\lambda)+\frac{ih\sqrt{\lambda}}{\lambda+a^2}m(\lambda)+\frac{h}{\lambda+a^2}m(\lambda)a=0,$$

$$\frac{i\sqrt{\lambda}}{\lambda+a^2}+m(\lambda)+\frac{ih\sqrt{\lambda}}{\lambda+a^2}m(\lambda)=0$$

$$m(\lambda)=\left(-\frac{i\sqrt{\lambda}}{\lambda+a^2}\right)\cdot\frac{1}{1+\frac{ih\sqrt{\lambda}}{\lambda+a^2}}=-\frac{i\sqrt{\lambda}}{\lambda+a^2+ih\sqrt{\lambda}}=$$
$$=-\frac{i\sqrt{\lambda}(\lambda+a^2)+h\lambda}{(\lambda+a^2)^2+h^2\lambda};$$

Demak,

$$\mathrm{Im}\{m(\lambda)\}=-\frac{(\lambda+a^2)\sqrt{\lambda}}{(\lambda+a^2)^2+h^2\lambda}\,,\qquad \lambda>0,$$

bo‘ladi.

Natija. Ushbu $m(\mathrm{z})=-\mathrm{ctg}\,\alpha+\int_{-\infty}^{\infty}\frac{d\rho(\lambda)}{z-\lambda}$, $(\sin\alpha\neq 0)$ tenglikga asosan quyidagi ifodaga kelamiz:

$$\rho'(\lambda)=\begin{cases}\dfrac{(\lambda+a^2)\sqrt{\lambda}}{\pi(\lambda+a^2)^2+h^2\lambda}, & \lambda>0,\\ 0, & \lambda\leq 0,\end{cases}$$

$$\rho(\lambda)=\begin{cases}\dfrac{1}{\pi}\displaystyle\int_0^{\lambda}\frac{(t+a^2)\sqrt{t}}{(t+a^2)^2+h^2t}dt\,, & \lambda>0,\\ 0, & \lambda\leq 0.\end{cases}$$

FOYDALANILGAN ADABIYOTLAR RO‘YXATI

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