O‘ZGARMAS KOEFFITSIYENTLI SIMMETRIK T-GIPERBOLIK SISTEMALARNI BIR O‘LCHOVLI SOHADA CHEKLI ELEMENTLAR USULI BILAN YECHISH

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ANNOTATSIYA

Ushbu ishda bir o‘lchovli o‘zgarmas koeffitsiyentli simmetrik *t*-giperbolik sistemalar uchun chekli elementlar usuli qaralgan. Simmetrik *t*-giperbolik sistemalar uchun qoyilgan aralash masalani approksimasiya qilishda, chekli elementlar usuli fazoviy sohani diskretizasiya qilishda foydalanilgan, vaqt boyicha diskretizasiya qilishda esa chekli ayirmali usuldan foydalanilgan. Simmetrik *t*-giperbolik sistemalar uchun qoyilgan aralash masalaning oshkormas ayirmali sxemasi qurilgan. Simmetrik *t*-giperbolik sistema uchun qoyilgan aralash masalani chekli elementlar usuli bilan sonli yechish algoritmi yaratilgan. Ushbu algoritm asosida simmetrik *t*-giperbolik sistema uchun qoyilgan aralash masalani chekli elementlar usuli bilan sonli yechadigan dastur yaratilgan. Model masalaning sonli hisoblashlar natijalari keltirilgan.

Kalit so‘zlar: chekli elementlar usuli,algoritm, aralash masala, giperbolik sistema, bazis funksiyalar, oshkormas ayirmali sxema.

ABSTRACT

This article discusses the finite element method for symmetric t-hyperbolic systems. In approximation of a mixed problem for symmetric t-hyperbolic systems, the finite element method is used to discretize in space, and the time discretization is done using finite difference. An implicit difference scheme of a mixed problem for symmetric t-hyperbolic systems was constructed. Created an algorithm for a numerical solution of a mixed problem for symmetric t-hyperbolic systems by finite elements.

Based on this algorithm, a program has been created for a numerical solution of a mixed problem for symmetric t-hyperbolic systems by the method of finite elements. The numerical calculation of the model problem is given.

Keywords: finite element method, algorithm, mixed problem, hyperbolic system, basic functions, implicit difference scheme.

1. Masalaning qo‘yilishi.

$\Omega=(\ell_1,\ell_2)$ bo‘lsin. $G=\{(t,x):t\in(0,T),\ x\in\Omega\}$ sohada

$$A\frac{\partial u}{\partial t}+B\frac{\partial u}{\partial x}+Cu=F(t,x) \tag{1}$$

simmetrik t-giperbolik sistemani

$$\begin{aligned} R_1u(t,\ell_1)&=g_1(t),\\ R_2u(t,\ell_2)&=g_2(t) \end{aligned} \tag{2}$$

chegaraviy va

$$u(0,x)=\psi(x),\quad x\in\Omega \tag{3}$$

boshlang‘ich shartlarni qanoatlantiruvchi u vektor-funksiyani topish talab qilingan bo‘lsin. (1)-(3) masalasi simmetrik t-giperbolik sistema uchun qo‘yilgan aralash masala deb nomlanadi[1,3].

Bu yerda A,B - $M\times M$ o‘lchamli simmetrik haqiqiy o‘zgarmas matritsalar; bundan tashqari A musbat aniqlangan matritsa; C - $M\times M$ o‘lchamli haqiqiy o‘zgarmas matritsa; R_1,R_2-mos to‘g‘riburchakli o‘zgarmas matritsalar bo‘lib, ularning ustunlar soni M ga, satrlari soni mos ravishda $A^{-1}B$ matritsaning musbat va manfiy xos qiymatlari soniga teng[1,3]; g_1,g_2-berilgan vektor-funksiyalar bo‘lib, ularning o‘lchamlari mos ravishda $A^{-1}B$ matritsaning musbat va manfiy xos qiymatlari soniga teng; $\psi(x)-$berilgan o‘lchami M ga teng bo‘lgan vektor-funksiya; $u(t,x)=(u_1,u_2,...,u_M)^T$ -no‘malum vektor-funksiya; $F(t,x)=(f_1,f_2,...,f_M)^T$ - berilgan vektor-funksiY.

2. (1)-(3) aralash masalaning chekli elementlar usuli yordamida olingan ayirmali sxemasi.

$[0,T]$ kesmani N_t bo‘lakka bo‘lamiz.

$$t_n=\tau\cdot n,(n=0,...,N_t),\tau=\frac{T}{N_t}.$$

$[\ell_1,\ell_2]$ kesmani N_x ta teng bo‘lakka bo‘lib ($x_i=\ell_1+hi$, $i=0,...,N_x$, $h=\frac{\ell_2-\ell_1}{N_x}$), aralash masalaning vaqt bo‘yicha har bir t_n qatlamdagi, $u_h(t_n,x)$ taqribiy yechimini

$u_h^n = u_h(t_n, x) = \sum_{i=0}^{N_x} u_i^n \varphi_i(x)$ ko‘rinishda izlaymiz. Bu yerda $\varphi_i(x)$ bazis funksiya[4], x_i tugunda uning qiymati 1 ga teng, qolgan tugunlarda esa uning qiymati 0 ga teng, $u_i^n = u(t_n, x_i) = (u_{1i}^n, u_{2i}^n, ..., u_{Mi}^n)^T$ vektor. (1) sistemada $\frac{\partial u}{\partial t}$ ni $\frac{u(t+\tau, x) - u(t,x)}{\tau}$ munosabat bilan almashtiramiz, $u(t,x)$ o‘rniga $u_h(t_n, x)$ qo‘yamiz va hosil bo‘lgan sistemaning har bir tenglamasini $\phi_0(x)$ bazis funksiyaga ko‘paytirib $[x_0, x_1]$ kesmada, $\phi_{N_x}(x)$ bazis funksiyaga ko‘paytirib $[x_{N_x-1}, x_{N_x}]$ kesmada va $\phi_i(x)$ bazis-funksiyalarga ko‘paytirib $[x_{i-1}, x_{i+1}], i = 1, ..., N_x - 1$ kesmalarda integrallaymiz. Bazis funksiyalar sifatida

$$\phi_i(x) = \begin{cases} \frac{x - x_{i-1}}{h}, & x \in (x_{i-1}, x_i); \\ \frac{x_{i+1} - x}{h}, & x \in (x_i, x_{i+1}); \qquad i = 1, ..., N_x\text{-}1 \\ 0, & x \notin (x_{i-1}, x_{i+1}); \end{cases} \tag{4}$$

$$\phi_0(x) = \begin{cases} \frac{x_1 - x}{h}, & x \in (x_0, x_1); \\ 0, & x \notin (x_0, x_1); \end{cases} \quad \phi_{N_x}(x) = \begin{cases} \frac{x - x_{N_x-1}}{h}, & x \in (x_{N_x\text{-}1}, x_{N_x}); \\ 0, & x \notin (x_{N_x\text{-}1}, x_{N_x}) \end{cases}$$

funksiyalarni oladigan bo‘lsak, (1) sistema uchun ushbu

$$AhL_0 u_0^{n+1} + \tau B\xi_0 u_0^{n+1} + \tau ChL_0 u_0^{n+1} = \frac{1}{2}\tau hF_0^{n+1} + AhL_0 u_0^n \tag{5}$$

$$AhL_{N_x} u_{N_x}^{n+1} + \tau B\xi_{N_x} u_{N_x}^{n+1} + \tau ChL_{N_x} u_{N_x}^{n+1} = \frac{1}{2}\tau hF_{N_x}^{n+1} + AhL_{N_x} u_{N_x}^n \tag{6}$$

$$AhLu_i^{n+1} + \tau B\xi u_i^{n+1} + \tau ChLu_i^{n+1} = \tau hF_i^{n+1} + AhLu_i^n$$
$$i = 1, ... N_x - 1 \tag{7}$$

ayirmali sxemani olamiz. Bu yerda $u_i^n = u(t_n, x_i)$, $F_i^n = F(t_n, x_i), i = 0, ..., N_x$ vektor-funksiyalarning approksimatsiyasi va quyidagi operatorlar kiritilgan:

$$L_0 = \frac{1}{3} + \frac{1}{6}\psi^{+1}, \ \xi_0 = \frac{\psi^{+1} - 1}{2}, L_{N_x} = \frac{1}{6}\psi^{-1} + \frac{1}{3}, \ \xi_{N_x} = \frac{1 - \psi^{-1}}{2},$$

$$\psi^{\pm 1} u_i^n = u_{i\pm 1}^n, \ L = \frac{1}{6}\psi^{-1} + \frac{2}{3} + \frac{1}{6}\psi^{+1}, \ \xi = \frac{\psi^{+1} - \psi^{-1}}{2}.$$

(1)-(3) masalaning chegaraviy va boshlang‘ich shartlarini quyidagicha approksimatsiya qilamiz:

$x=\ell_1$ da $\qquad R_1 u_0(t_{n+1}) = g_1(t_{n+1})$, (8)

$x=\ell_2$ da $\qquad R_2 u_{N_x}(t_{n+1}) = g_2(t_{n+1})$, (9)

$$u_i(0)=\psi(x_i) \quad i=\overline{0,N_x}\ . \qquad (10)$$

Aralash masalaning $t_n (n=0,1,2,...)$ qatlamgacha taqribiy yechimi topilgan deb, t_{n+1} qatlamdagi taqribiy yechimini topish uchun u_i^{n+1}, $(i=0,...,N_x)$ vektorlarning komponentalariga nisbatan (7)-(9) chiziqli tenglamalar sistemasini hosil qilamiz. Chiziqli tenglamalar sistemasi yopiq bo‘lishi maqsadida $x=\ell_1$ da $u(t,x)$ vektor-funksiyaning chegaraviy shart qo‘yilmagan komponentalari uchun (5) ayirmali tenglamalar sistemasidan ularga mos keluvchi tenglamalarni olamiz. Masalan, $u(t,x)$ vektor-funksiyaning $j-$komponentasiga chegaraviy shart qo‘yilmagan bo‘lsa, (5) ayirmali tenglamalar sistemasidan $j-$tenglamani olamiz. $x=\ell_2$ da esa $u(t,x)$ vektor-funksiyaning chegaraviy shart qo‘yilmagan komponentalari uchun (6) ayirmali tenglamalar sistemasidan ularga mos keluvchi tenglamalarni olamiz. Shunday qilib yopiq chiziqli tenglamalar sistemasini hosil qilamiz. Bu chiziqli tenglamalar sistemasi bosh elementlar usuli bilan yechilgan.

3. Chekli elementlar usuli yordamida olingan ayirmali sxemaning turg‘unligi

Chekli elementlar usuli yordamida hosil qilingan ayirmali sxemaning turg‘unligini isbotlash qulay bo‘lishi uchun A matritsani birlik matritsa deb hisoblaymiz. Ω sohani chekli, ichki umumiy nuqtalarga ega bo‘lmagan elementlarga (kesmalarga) bo‘lamiz .Elementni (kesmani) K harfi bilan belgilaymiz. U holda $\Omega=\bigcup_{K\subset\Omega} K$. $[0,T]$ kesmani N_t bo‘lakka bo‘lamiz.

$$t_k=\tau\cdot k,(k=0,...,N_t),\tau=\frac{T}{N_t}$$

Chekli elementlar usuli yordamida hosil qilingan ayirmali sxemaning turg‘unligini isbotlash uchun (1)-(3) aralash masala yagona yechimga ega va quyidagi shartlar

$$(Bu,u)\big|_{x=\ell_2}-(Bu,u)\big|_{x=\ell_1}\ge 0, \qquad (11)$$

$$C+C^T\ge 0 \qquad (12)$$

bajariladi deb faraz qilamiz.

Quyidagi sistemani ko‘rib chiqamiz.

$$Lu\equiv\tau B\frac{\partial u}{\partial x}(t,x)+(I+\tau\cdot C)u(t,x)=u(t-\tau,x)+\tau\cdot F(t,x). \qquad (13)$$

Bu yerda I birlik matritsa.

Bichiziqli forma kiritamiz

$$a(u,v)\equiv(Lu,v)_K. \tag{14}$$

Bu yerda

$$(u,v)_K=\int_K u\cdot v dx. \tag{15}$$

Har bir element K da u yechimni $u_h\in(P_m(K))^M$ orqali approksimatsiya qilamiz. Bu yerda $(P_m(K))^M=P_m(K)\times P_m(K)\times\cdots\times P_m(K)$, $P_m(K)$ koeffitsiyentlari t ga bog‘liq bo‘lgan darajasi $\leq m$, K da aniqlangan ko‘phadlar to‘plami.

U holda, har bir element K da, quyidagi tenglik o‘rinli bo‘ladi.

$$\left(\tau B\frac{\partial u_h}{\partial x}(t_k,x)+(I+\tau\cdot C)u_h(t_k,x),v_h\right)_K=(u_h(t_{k-1},x)+\tau\cdot F_h(t_k,x),v_h)_K,\forall v_h\in(P_m(K))^M. \tag{16}$$

Agar $v_h(x)\in P_m(K)$ bazis funksiya[4] bo‘lsa (16) tenglikdan, element K da (1) sistemaning oshkormas ayirmali sxemasini hosil qilamiz.

Sxema turg‘unligini isbotlash uchun quyidagi operatorni kiritamiz

$$L^*v\equiv v-\tau B\frac{\partial v}{\partial x}(t,x)+\tau C^T v(t,x). \tag{17}$$

Quyidagi belgilashlarni kiritamiz:

Agar $K=[\alpha,\beta]$ kesma bo‘lsa, u holda $\Gamma(K)=\{\alpha;\beta\}$ bo‘ladi.

$$(Du\cdot v)\big|_{\Gamma(K)}=(Du\cdot v)\big|_{x=\beta}+(Du\cdot v)\big|_{x=\alpha}=(Bu\cdot v)\big|_{x=\beta}-(Bu\cdot v)\big|_{x=\alpha}. \tag{18}$$

Bu yerda $D=nB$, n K kesmaga birlik tashqi normal.

Lemma-1

$$a(u,v)=\left(u,L^*v\right)_K+\tau(Du\cdot v)\big|_{\Gamma(K)-\Gamma^*(K)}+\tau(Du\cdot v)\big|_{\Gamma^*(K)}. \tag{19}$$

Bu yerda $\Gamma^*(K)=\Gamma(K)\cap\Gamma(\Omega)$.

Isbot.

$$a(u,v)\equiv(Lu,v)_K=\left(\tau B\frac{\partial u}{\partial x}+(I+\tau\cdot C)u,v\right)_K=$$

bo‘laklab integrallashni qo‘llaymiz.

$$=\tau\left((Du\cdot v)\big|_{\Gamma(K)}-\left(B\frac{\partial v}{\partial x},u\right)_K\right)+\left((I+\tau\cdot C^T)v,u\right)_K=$$

$$=\left(v-\tau B\frac{\partial v}{\partial x}+\tau C^T v,u\right)_K+\tau(Du\cdot v)\big|_{\Gamma(K)-\Gamma^*(K)}+\tau(Du\cdot v)\big|_{\Gamma^*(K)}. \tag{20}$$

Lemma-1 isbot qilindi.

Lemma-2

$$a(u,u)\geq(u,u)_K+\frac{\tau}{2}(Du\cdot v)\big|_{\Gamma(K)-\Gamma^*(K)}+\frac{\tau}{2}(Du\cdot u)\big|_{\Gamma^*(K)}. \qquad (21)$$

Isbot.

Bir nechta sodda amallarni bajaramiz

$$\left(u,L^*u\right)_K=\left(u-\tau B\frac{\partial u}{\partial x}+\tau C^T u,u\right)_K=-\left(u+\tau B\frac{\partial u}{\partial x}+\tau Cu,u\right)_K+\left(2u+\tau Cu+\tau C^T u,u\right)_K=$$

$$=-a(u,u)+\left((2I+\tau(C+C^T))u,u\right)_K. \qquad (22)$$

(19) va (22) yordamida (12) shartni e'tiborga olib,quyidagini hosil qilamiz.

$$a(u,u)=\frac{1}{2}\left((2I+\tau(C+C^T))u,u\right)_K+\frac{\tau}{2}(Du\cdot u)\big|_{\Gamma(K)-\Gamma^*(K)}+$$
$$+\frac{\tau}{2}(Du\cdot u)\big|_{\Gamma^*(K)}\geq(u,u)_K+\frac{\tau}{2}(Du\cdot u)\big|_{\Gamma(K)-\Gamma^*(K)}+\frac{\tau}{2}(Du\cdot u)\big|_{\Gamma^*(K)}. \qquad (23)$$

Teorema.

Taqribiy yechim $u_h\in(P_m(K))^M$ K da bir qiymatli aniqlangan va barcha $k\leq\frac{T}{\tau}$ larda quyidagi tengsizlik o'rinli

$$\left\|u_h^k\right\|_\Omega^2\leq e^T\left\|\psi\right\|_\Omega^2+(T+1)(e^T-1)F \qquad (24)$$

Bu yerda norma $\|u\|_\Omega=\sqrt{\int_\Omega u\cdot udx}$, $F=\max_k\left\|F_h^k\right\|_\Omega^2$.

Isbot. Vaqt bo'yicha $t_{k-1}=\tau(k-1)$ qatlamgacha taqribiy yechim $u_h^{k-1}\in(P_m(K))^M$ qiymatlari topilgan va $v_h(x)\in P_m(K)$ bazis funksiya tanlangan, deb faraz qilsak $t_k=\tau k$ qatlamda, (16) tenglikdan, u_h^k ga nisbatan quyidagi chiziqli algebraik sistemani olamiz.

$$\left(\tau B\frac{\partial u_h}{\partial x}(t_k,x)+(I+\tau\cdot C)u_h(t_k,x),v_h\right)_K=(u_h(t_{k-1},x)+\tau\cdot F_h(t_k,x),v_h)_K. \qquad (25)$$

Oxirgi (25) chiziqli algebraik sistemani matritsa ko'rinishida yozamiz.

$$\overline{A}_h\overline{u}_h=\overline{b}_h. \qquad (26)$$

Agar $\overline{A}_h\overline{u}_h=0$ sistema trivial yechimga $\overline{u}_h\equiv0$ ega ekanligini ko'rsata olsak, u_h yechim bir qiymatli aniqlanganligi kelib chiqadi.Bu $\forall K$ da

$$\left(\tau B\frac{\partial u_h}{\partial x}(t_k,x)+(I+\tau\cdot C)u_h(t_k,x),v_h\right)_K=$$
$$=0,\forall v_h\in(P_m(K))^M \qquad (27)$$

bo'lsa $u_h=0$ ekanligini ko'rsatish bilan teng kuchli.

$v_h=u_h$ olib, (27) ning chap tomoni $a(u_h,v_h)$ ekanini e'tiborga olib 2-lemmadan quyidagi tengsizlikni olamiz.

$$\left(u_h,u_h\right)_K+\frac{\tau}{2}(D_hu_h\cdot u_h)\Big|_{\Gamma(K)-\Gamma^*(K)}+\frac{\tau}{2}(D_hu_h\cdot u_h)\Big|_{\Gamma^*(K)}\le 0. \tag{28}$$

U holda $K\in\Omega_h$ ixtiyoriy element bo‘lgani uchun (28) tengsizlikdan quyidagi tengsizlik kelib chiqadi.

$$\left(u_h,u_h\right)_\Omega+\frac{\tau}{2}\sum_{K\in\Omega}(D_hu_h\cdot u_h)\Big|_{\Gamma(K)-\Gamma^*(K)}+\frac{\tau}{2}(D_hu_h\cdot u_h)\Big|_{\Gamma(\Omega)}\le 0. \tag{29}$$

Ko‘rinib turibdiki

$$\sum_{K\in\Omega}(D_hu_h\cdot u_h)\Big|_{\Gamma(K)-\Gamma^*(K)}\equiv 0. \tag{30}$$

(30) ni va (11) shartni e’tiborga olib, (29) dan quyidagi tengsizlik kelib chiqadi.

$$\left(u_h,u_h\right)_\Omega\le 0. \tag{31}$$

Oxirgi tengsizlikdan $u_h\equiv 0$. 2-lemmaga ko‘ra (21) tengsizlikdan quyidagi tengsizlikni olamiz.

$$2\left(u_h^k,u_h^k\right)_K+\tau(D_hu_h^k\cdot u_h^k)\Big|_{\Gamma(K)-\Gamma^*(K)}+\tau(D_hu_h^k\cdot u_h^k)\Big|_{\Gamma^*(K)}\le 2(u_h^{k-1}+\tau\cdot F_h^k,u_h^k)_K. \tag{32}$$

Bu yerda $u_h^k=u_h(t_k,x), u_h^{k-1}=u_h(t_{k-1},x), F_h^k=F_h(t_k,x)$. (32) tengsizlikni quyidagi ko‘rinishda yozamiz.

$$\begin{aligned}&2\left\|u_h^k\right\|_K^2+\tau(D_hu_h^k\cdot u_h^k)\Big|_{\Gamma(K)-\Gamma^*(K)}+\tau(D_hu_h^k\cdot u_h^k)\Big|_{\Gamma^*(K)}\le 2(u_h^{k-1}+\tau\cdot F_h^k,u_h^k)_K\le\\&\left\|u_h^{k-1}+\tau\cdot F_h^k\right\|_K^2+\left\|u_h^k\right\|_K^2.\end{aligned} \tag{33}$$

Oxirgi tengsizlikdan quyidagi tengsizlikni olamiz:

$$\begin{aligned}&\left\|u_h^k\right\|_K^2+\tau(D_hu_h^k\cdot u_h^k)\Big|_{\Gamma(K)-\Gamma^*(K)}+\tau(D_hu_h^k\cdot u_h^k)\Big|_{\Gamma^*(K)}\le\left\|u_h^{k-1}+\tau\cdot F_h^k\right\|_K^2\le\\&\le\left(\left\|u_h^{k-1}\right\|_K+\tau\left\|F_h^k\right\|_K\right)^2\le(\tau+1)\left\|u_h^{k-1}\right\|_K^2+(\tau^2+\tau)\left\|F_h^k\right\|_K^2.\end{aligned} \tag{34}$$

(30) ayniyatni va (11) shartni e’tiborga olib (34) tengsizlikdan quyidagi tengsizlik o‘rinli ekanligi kelib chiqadi.

$$\left\|u_h^k\right\|_\Omega^2\le(\tau+1)\left\|u_h^{k-1}\right\|_\Omega^2+(\tau^2+\tau)\left\|F_h^k\right\|_\Omega^2. \tag{35}$$

Oxirgi tengsizlikdan

$$\left\|u_h^k\right\|_\Omega^2\le e^T\left\|\psi\right\|_\Omega^2+(T+1)(e^T-1)F \tag{36}$$

kelib chiqadi. Bu yerda $F=\max_k\left\|F_h^k\right\|_\Omega^2$. Teorema isbot qilindi.

Ushbu teorema CHEU bilan olingan (1)-(3) masalaning oshkormas sxemasi (11) va (12) shartlar bajarilganda turg‘un ekanligini ko‘rsatadi.

Ushbu algoritm asosida, Delphi-7 dasturlash tilida, sxema turg‘unligining yetarli shartlari bo‘lgan (11) va (12) shartlarni tekshirib shartlar bajarilmagan holatda bajarilmaganligi haqida ma’lumot beradigan, (1)-(3) masalani sonli yechadigan va yechim grafigini chizadigan dastur yaratilgan.

4. Olingan natijalar.

1-masala[2].

$\Omega=\{x:0<x<2\pi\}$ bo‘lsin. $G=\{(t,x):t\in(0,T),\ x\in\Omega\}$ sohada

$$u_t+\begin{pmatrix}0&-1\\-1&0\end{pmatrix}u_x+\begin{pmatrix}0&0\\0&\beta\end{pmatrix}u=\begin{pmatrix}0\\\varphi\end{pmatrix}$$

sistemani

$x=0$ va $x=2\pi$ da

$$u_2=0$$

chegaraviy shartni va

$t=0$ da

$$u_1=0, u_2=\sqrt{2}\sin x$$

boshlang‘ich shartlarni qanoatlantiruvchi u vektor-funksiyani topish talab qilingan bo‘lsin. Bu yerda $\beta=1$ va $\varphi=\sqrt{2}\sin x\cos\sqrt{2}t-\sin x\sin\sqrt{2}t$.

$t>0$ uchun bunday aralash masalaning aniq yechimi

$$u_1=\cos x\sin\sqrt{2}t, u_2=\sqrt{2}\sin x\cos\sqrt{2}t$$

bo‘ladi.

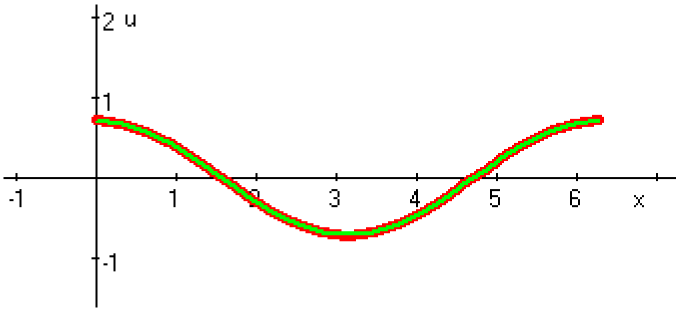
Berilgan aralash masalada teorema shartlarining bajarilishini tekshirib ko‘rish qiyin emas.

Pastdagi jadvalda $\|u\|_\Omega=\sqrt{\int_\Omega u\cdot u dx}$ normada, x va vaqt bo‘yicha bo‘laklar soni har xil bo‘lganda, $T=5$ dagi aniq yechim bilan chekli elementlar usuli orqali olingan yechim farqi $\|u-v\|_\Omega$ qanday bo‘lgani keltirilgan. Bu yerda $v=(v_1,v_2)^T$ chekli elementlar usuli orqali olingan yechim.

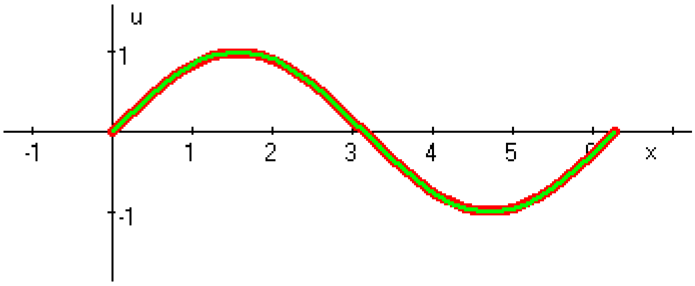
N_t	$N_x=10$	$N_x=20$	$N_x=40$	$N_x=80$	$N_x=160$
10	0.9454	0.9910	1.0025	1.0054	1.0061
20	0.5554	0.6071	0.6209	0.6244	0.6253
40	0.2851	0.3336	0.3489	0.3528	0.3538
80	0.1440	0.1683	0.1836	0.1878	0.1889
160	0.1114	0.0790	0.0922	0.0964	0.0975
320	0.1235	0.0378	0.0443	0.0483	0.0494
640	0.1166	0.0281	0.0202	0.0236	0.0247
1200	0.1039	0.0303	0.0101	0.0120	0.0130
2400	0.1014	0.0301	0.0070	0.0055	0.0063

$T = 5$ **da aniq yechim bilan taqribiy yechim farqi** $\|u - v\|_{\Omega}$.

1 va 2 rasmlarda taqribiy yechim grafigi qizil rangda va aniq yechim grafigi yashil rangda tasvirlangan.



1-rasm. v_1 va u_1 yechim grafiklari. $T = 5, N_t = 640, N_x = 40$.



2-rasm. v_2 va u_2 yechim grafiklari. $T = 5,\ N_t = 640, N_x = 40$.

XULOSA

Simmetrik *t*-giperbolik sistemalar uchun qoyilgan aralash masala uchun oshkormas ayirmali sxema qurildi. Ma'lum bir shartlar bajarilganda simmetrik *t*-giperbolik sistema uchun chekli elementlar usuli turg'unligi isbotlandi.Simmetrik *t*-giperbolik sistema uchun qoyilgan aralash masalani chekli elementlar usuli bilan sonli yechish algoritmi yaratildi. Ushbu algoritm asosida simmetrik *t*-giperbolik sistema uchun qoyilgan aralash masalani chekli elementlar usuli bilan sonli yechadigan dastur yaratildi. Dasturning to'g'ri ishlashi modelli masalada tekshirildi.

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