IKKINCHI TARTIBLI GRONUOLL CHEGARALANISHLI BOSHQARUVLAR UCHUN TUTISH MASALASI

Polvanov Rashid Raximjanovich,
G‘ayniddinov Shayxislom Tolibjon o‘g‘li,
Namangan davlat universiteti ”Algebra va matematika o‘qitish metodikasi” kafedrasi dotsenti, Namangan davlat universiteti ”Algebra va matematika o‘qitish metodikasi” kafedrasi o‘qituvchisi.

Annotatsiya. *Ushbu maqolada boshqaruvlar Granoull chegaralanishga ega holda ikkinchi tartibli differensial o‘yinlar uchun tutish masalasi o‘rganiladi. Bunda quvlovchi uchun parallel quvish strategiyasi quriladi va uning yordamida tutish masalasi uchun yetarli shartlar keltiriladi.*

Kalit so‘zlar: *Differensial o‘yin, geometrik chegaralanish, parallel quvish strategiyasi, quvlovchi, qochuvchi, tezlanish, Granoull chegaralanishli.*

ЗАДАЧА СОХРАНЕНИЯ ГРОНУЛА ВТОРОГО ПОРЯДКА ДЛЯ ГРАНИЧНЫХ УПРАВЛЕНИЙ

Полванов Рашид Рахимжанович,
Гайниддинов Шайхислам Талибжонович,
Наманганский государственный университет, доцент кафедры «Методики преподавания алгебры и математики», Преподаватель кафедры «Методики преподавания алгебры и математики» Наманганского государственного университета.

Аннотация. *В работе рассматривается дифференциальная игра второго порядка при геометрических ограничениях на управления игроков. При этом предлагается стратегия параллельного преследования для преследователя и при помощи этой стратегии решается задача преследования.*

Ключевые слова: *дифференциальная игра, геометрическое ограничение, стратегия параллельного преследования, преследователь, убегающий, ускорения.*

A SECOND-ORDER GRONOULL CONSERVATION PROBLEM FOR BOUNDARY CONTROLS

Polvanov Rashid Rakhimjanovich,
Gayiniddinov Sheikhislam Talibjan ugli,
Associate professor of the "Algebra and mathematics teaching methodology" department of Namangan State University, teacher of the "Algebra and mathematics teaching methodology" department of Namangan State University.

Abstract: *This paper investigates the problem of holding for second-order differential games with control Granoull boundedness. In this case, a parallel pursuit strategy is constructed for the pursuer and with its help, sufficient conditions for the capture problem are given.*

Key words: *Differential game, geometric boundedness, parallel pursuit strategy, chaser, escaper, acceleration, Granoull bounded.*

Let **P** and **E** objects with opposite aim be given in $\boldsymbol{R}^n$ space and their movements based on the following differential equations and initial conditions

$$\mathbf{P}:\ \ddot{x}=u,\ x_1-kx_0=0,\ |u(t)|^2\le\rho^2+2l\int_0^t|u(s)|^2\,ds, \tag{1}$$

$$\mathbf{E}:\ \ddot{y}=v,\ y_1-ky_0=0,\ |v(t)|^2\le\sigma^2+2l\int_0^t|v(s)|^2\,ds, \tag{2}$$

where $x,y,u,v\in\boldsymbol{R}^n$; x – a position of **P** object in $\boldsymbol{R}^n$ space, $x_0=x(0),\ x_1=\dot{x}(0)$ – its initial position and velocity respectively at $t=0$; u – a controlled acceleration of the pursuer, mapping $u:[0,\infty)\to\boldsymbol{R}^n$ and it is chosen as a measurable function with respect to time; we denote a set of all measurable functions $u(\cdot)$ such that satisfies the condition $|u(t)|^2\le\rho^2+2l\int_0^t|u(s)|^2\,ds$ by G_P. y – a position of **E** object in $\boldsymbol{R}^n$ space, $y_0=y(0),\ y_1=\dot{y}(0)$ – its initial position and velocity respectively at $t=0$; v – a controlled acceleration of the evader, mapping $v:[0,\infty)\to\boldsymbol{R}^n$ and it is chosen as a measurable function with respect to time; we denote a set of all measurable functions $v(\cdot)$ such that satisfies the condition $|v(t)|^2 \le \sigma^2 + 2l\int_0^t |v(s)|^2 ds$ by G_E.

Definition 1. For a trio of $(x_0, x_1, u(\cdot)), u(\cdot) \in G_P$, the solution of the equation (1), that is, $x(t) = x_0 + x_1 t + \int_0^t \int_0^s u(\tau) d\tau ds$ is called a trajectory of the pursuer on interval $t \ge 0$.

Definition 2. For a trio of $(y_0, y_1, v(\cdot)), v(\cdot) \in G_E$, the solution of the equation (2), that is, $y(t) = y_0 + y_1 t + \int_0^t \int_0^s v(\tau) d\tau ds$ is called a trajectory of the evader on interval $t \ge 0$.

Definition 3. The pursuit problem for the differential game (1) - (2) is called to be solved if there exists such control function $u^*(\cdot) \in \boldsymbol{G_p}$ of the pursuer for any control function $v(\cdot) \in \boldsymbol{G_E}$ of the evader and the following equality is carried out at some finite time t^*

$$x(t^*) = y(t^*). \tag{3}$$

Definition 4. For the problem (1)-(2), time T is called a guaranteed pursuit time if it is equal to an upper boundary of all the finite values of pursuit time t^* which the equality (3) is true.

Definition 5. For the differential game (1) - (2), the following function is called П-strategy of the pursuer ([3]-[4]):

$$u(v) = v - \lambda(v)\xi_0, \tag{4}$$

where $\xi_0 = \frac{z_0}{|z_0|}$, $\lambda(v) = (v, \xi_0) + \sqrt{(v, \xi_0)^2 + \delta e^{2lt}}$, $\delta = \rho^2 - \sigma^2 \ge 0$, (v, ξ_0) is a scalar multiplication of vectors v and ξ_0 in the space $\boldsymbol{R^n}$.

Lemma 1 (Granwoll). Suppose, let a mapping $\varphi(t): [0, \infty) \to \boldsymbol{R^n}$ be bounded, nonnegative and measurable function. Moreover, $l \ge 0$ and $\rho > 0$ are constant and for the given if an inequality $|\varphi(t)|^2 \le \rho^2 + 2l\int_0^t |\varphi(s)|^2 ds$ is carried out, then a relation $\varphi(t) \le \rho e^{lt}$ is always true.

Lemma 2 . If $\rho \geq \sigma$, then the following inequality is true for the function $\lambda\left(v,\xi_0\right)$:

$$e^{lt}(\rho-\sigma) \leq \lambda\left(v,\xi_0\right) \leq e^{lt}(\rho+\sigma).$$

Theorem. If for the second order differential game (1) – (2) with Granwoll constraint a condition $\rho > \sigma$ is true, then the pursuit problem is solved by П-strategy (4) on interval $(0,t)$ and an approach function between the objects becomes as follows:

$$f(l,t,|z_0|,\rho,\sigma,k) = |z_0|(kt+1) - \frac{\rho-\sigma}{l^2}e^{lt} + \frac{\rho-\sigma}{l^2} + \frac{\rho-\sigma}{l}t$$

Proof. Suppose, let the pursuer choose a strategy in the form (4) when the evader chooses any control function $v(\cdot) \in \boldsymbol{G_E}$. Then according to the equations (1) and (2) we define the following Caratheodory's equation

$$\ddot{z} = -\lambda\left(v(t)\right)\xi_0, \qquad \dot{z}(0) - kz(0) = 0,$$

Hence the following solution will be found by the given initial conditions

$$z(t) = z_0\left(kt+1\right) - \xi_0 \int_0^t\int_0^s \lambda\left(v(\tau),\xi_0\right)d\tau ds$$

or

$$\left|z(t)\right| = |z_0|\left(kt+1\right) - \int_0^t\int_0^s ((v,\xi_0) + \sqrt{(v,\xi_0)^2 + \delta e^{2lt}})d\tau ds.$$

We form the following inequalities in relation to **Lemma 1**

$$\left|z(t)\right| \leq |z_0|\left(kt+1\right) - \int_0^t\int_0^s e^{l\tau}(\rho-\sigma)d\tau ds \Rightarrow$$

$$|z(t)| \leq |z_0|(kt+1) - \frac{\rho-\sigma}{l^2}e^{lt} + \frac{\rho-\sigma}{l^2} + \frac{\rho-\sigma}{l}t$$

If we say $f(l,t,|z_0|,\rho,\sigma,k) = |z_0|(kt+1) - \frac{\rho-\sigma}{l^2}e^{lt} + \frac{\rho-\sigma}{l^2} + \frac{\rho-\sigma}{l}t$ (5),

then define a positive solution t^* such that the function (5) equals to zero

$$\frac{\rho-\sigma}{l^2}e^{lt} = |z_0|(kt+1) + \frac{\rho-\sigma}{l^2} + \frac{\rho-\sigma}{l}t.$$

We will form the following equality by simplifying the latest relation

$$e^{lt} = t\left(\frac{|z_0|kl^2}{\rho-\sigma} + l\right) + \frac{|z_0|l^2}{\rho-\sigma} + 1$$

where $A=\frac{|z_0|kl^2}{\rho-\sigma}+l$, $B=\frac{|z_0|l^2}{\rho-\sigma}+1$, $B>1$. Therefore, we have the following equation

$$e^{lt}=At+B \qquad (6)$$

In order to define a pursuit time we will consider some cases of the equation (5).

1. Let be $A<0 \Rightarrow k<\frac{\sigma-\rho}{|z_0|l}$. Then the equation (5) has a unique positive solution t^* and this solution is a pursuit time (Fig-1).

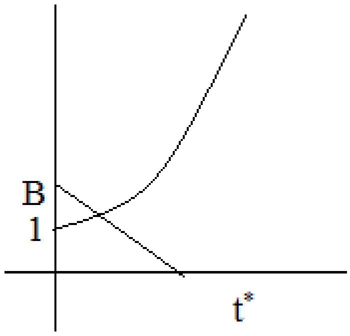

Figure-1

2. Let be $A=0 \Rightarrow k=\frac{\sigma-\rho}{|z_0|l}$. Then a solution of the equation (5) is $t^*=\frac{\ln\left(\frac{|z_0|l^2}{\rho-\sigma}+1\right)}{l}$, and this solution is a pursuit time (Fig-2).

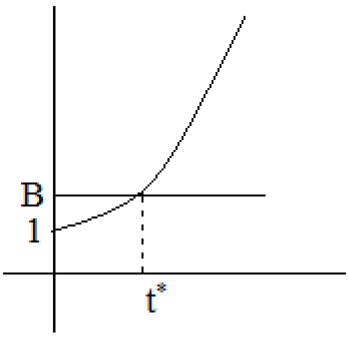

Figure-2

3. Let be $A>0 \Rightarrow k>\frac{\sigma-\rho}{|z_0|l}$. Then the equation (5) has a positive solution t^* and this solution is a pursuit time (Fig-3).

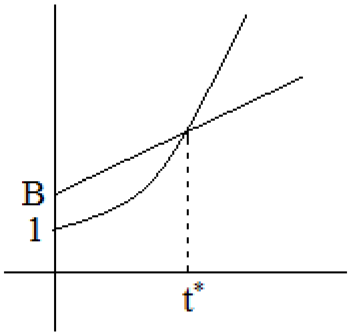

Figure-3

In conclusion, the relation (3) is true in all values of interval $t \geq 0$ according to the inequality $|z(t)| \leq f(k,t,\rho,\sigma,l,|z_0|)$ and properties of (5), i.e., the evasion problem is solved. Proved.

References.

1. Isaacs R. Differential games. John Wiley and Sons, New York, 1965 .
2. Nahin P.J. Chases and Escapes: The Mathematics of Pursuit and Evasion. Princeton University Press, Princeton, 2012 .
3. Azamov A.A., Samatov B.T. The П-Strategy: Analogies and Applications. The Fourth International Conference Game Theory and Management, St. Petersburg, Russia: 2010, p. 33-47.
4. Samatov B.T. The Pursuit- Evasion Problem under Integral-Geometric constraints on Pursuer controls. Automation and Remote Control, Pleiades Publishing, Ltd. New York: 2013, 74(7), p. 1072-1081.