MATHEMATICAL MODEL OF DYNAMIC PROCESSES IN HYDRAULIC SYSTEMS

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Abstract. This article presents the developed mathematical model of a hydraulic drive with distributed parameters, taking into account wave processes, the flow regime of the liquid (laminar, turbulent), extreme operating conditions and a model of thermal processes in a flow hydraulic system. These models allow us to study the dynamic characteristics in the hydraulic drive, to determine the changes in the temperature of the liquid depending on the level of circulation in the flow system.

Keywords: mathematical model, hydraulic drive, distributed parameters, hydraulic drives, laminar, turbulent.

INTRODUCTION

The general trend in the calculations of hydraulic drives is to complicate the mathematical models of drives, taking into account an increasing number of factors that affect the accuracy and reliability of the results obtained **[1]**. The mathematical model of the hydraulic drive can be obtained on the basis of the Navier-Stokes equation, the continuity equation of the flow, the heat balance equation, the equation that establishes the dependence of the viscosity, density and modulus of bulk elasticity of the liquid on temperature and pressure. The initial and boundary conditions are added to these equations. In the general case, the system under study turns out to be a nonlinear system, and the calculation of such dynamic systems leads to significant difficulties. Therefore, the mathematical model can be simplified by averaging the variables of pressure, velocity, and temperature over the flow section of the working medium [10]

Depending on these assumptions, three basic models have become common in the mathematical description of transients in hydraulic drives [1, 16]:

In the first model, the fluid is considered as a system with distributed parameters (elasticity, mass, and resistance).

In the second model, the liquid is considered compressible and concentrated, usually in one or two volumes of small extent (a system with concentrated parameters taking into account the flexibility of the hydraulic drive elements).

The third model is the simplest and is based on the fact that the transients in the hydraulic drive are described without taking into account the pliability of the liquid and its elements (pipelines, hoses, cylinders, etc.). This model in many cases does not allow us to give a reasonable assessment of the quality of the transition process of the hydraulic drive.

METHOD

In the case of choosing a model with distributed parameters, the equation of motion of a viscous compressible fluid in an elastic cylindrical tube of circular cross-section takes the form [2].

$$\frac{\partial V}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x} - \frac{\tau_{ts}}{\rho l} \quad (1)$$

$$\frac{\partial p}{\partial t} = -\left[\frac{E_L \delta_p E_p}{E_p \delta_p + d_p E_L}\right]\frac{\partial V}{\partial x} \quad (2)$$

Where p and V – fluid pressure and velocity; t – time; x – coordinate along the highway axis; ρ and E_L – density modulus of bulk liquid; d_p, δ_p, E_p – accordingly, the diameter, wall thickness, elastic modulus of the pipeline material.

Equation (1), in addition to p and V, includes the nonstationary tangential stress of viscous friction τ_{ts}. To obtain a closed system of equations, it is necessary to associate τ_{ts} with p or with V.

The value of τ_{ts} can be calculated from the ratio known from hydraulics [3]

$$\tau_{ts} = \lambda \frac{\rho l}{2d} V^2 \quad (3)$$

where λ– coefficient of friction loss; l and d – pipe length and diameter.

In the laminar mode (Re < 2300), the value $\tau_{ts}$is determined based on the Poiseuille equation [3]:

$$\tau_{ts} = \lambda \frac{8\pi \nu \rho l}{f} V \quad (4)$$

In the turbulent mode (Re ≥ 2300):

$$\tau_{ts} = \lambda \frac{0,443 \rho l}{f^{1/2}} V^2 \quad (5)$$

The coefficient of hydraulic friction λ can depend on two dimensionless parameters: the Reynolds number *Re=Vd/ν* and the relative roughness $\varepsilon = k/d$, where k is the roughness coefficient, hence λ *=f (Re, k/d)*[23].

The first systematic experiments to identify the nature of the dependence of λ on the number *Re* and *k/d* were conducted by I. Nikuradze [4]. As a result of the experiments of Nikuradze and other researchers on the resistance of pipelines, various empirical formulas were proposed for determining the coefficient of hydraulic friction.

For hydraulically smooth pipes, the Blasius formula is widely used (for 2300 <Re<8000) [3]

$$\lambda = \frac{0{,}3164}{\mathrm{Re}^{0{,}25}} \tag{6}$$

The dependence proposed by Nikuradze [5] is also applied

$$\lambda = 0{,}0032 + \frac{0{,}221}{\mathrm{Re}^{0{,}237}} \tag{7}$$

For quite rough pipes, the Shifrinson formula is used [6, 20]:

$$\lambda = 0{,}11(k/d)^{0{,}25} \tag{8}$$

A. D. Altshud [7] suggests the dependence λ in the following for

$$\lambda = 0{,}11(k/d + 68/\mathrm{Re})^{0{,}25} \tag{9}$$

At the limits, this formula passes into the well-known formulas of Blasius (for *Re(k/d) < 10*) and Schifrinson (for *Re(k/d) >500*).

Thus, for a more realistic determination of the coefficient λ, you can use the given formulas for the sections depending on the number Re:

$$\lambda = \begin{cases} 64/\mathrm{Re} & at\ \mathrm{Re} \le 2300 \\ 0{,}3164/\mathrm{Re}^{0{,}25} & at\ 2300 < \mathrm{Re} \le 8000 \\ 0{,}11(\frac{k}{d} + 68/\mathrm{Re})^{0.25} & at\ 8000 < \mathrm{Re} \le 60000 \\ 0{,}11(\frac{k}{d})^{0.25} & at\ \mathrm{Re} < 60000 \end{cases} \tag{10}$$

Such a functional dependence λ allows us to obtain accurate results of calculating the transition process, but this complicates the differential equation, and therefore it must be solved in sections (and the solution of this equation, taking into account the use of difference schemes, becomes problematic).

As shown by the studies conducted by Metlyuk N. F. and Avtushko V. P. [11], the complex effect of the Reynolds number *Re* and the relative roughness ε of the highway on the coefficient λ of the friction resistance can be taken into account with sufficient accuracy for practical calculations if the dependence λ *=f (Re, ε)* is approximated by a hyperbolic function of the form

$$\lambda = 70/\mathrm{Re} + k_{\varepsilon} \tag{11}$$

k_ε – the approximation coefficient, the value of which depends on the relative roughness e in hydraulic lines.

Table 1. Below are the values k_ε

ε	0,0001	0,001	0,002	0,005	0,010
k_ε	0,0186	0,022	0,026	0,031	0,038

Then the value of τ_{ts}, taking into account (3) and (11), is determined by the expression

$$\tau_{ts} = 27{,}5\frac{\nu\rho l}{f}V + 0{,}443\frac{k_\varepsilon \rho l}{f}V^2 \quad (12)$$

Expression (12) makes it possible to automatically take into account the flow mode of the fluid in the main line for any variations in the parameters of the hydraulic drive.

Substituting (12) into equation (1) we get

$$\frac{\partial V}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x} - 27{,}5\frac{\nu}{f}V - 0{,}443\frac{k_\varepsilon}{f^{1/2}}V^2 \quad (13)$$

$$\frac{\partial p}{\partial t} = -\left[\frac{E_L \delta_p E_p}{E_p \delta_p + d_p E_L}\right]\frac{\partial V}{\partial x} \quad (14)$$

or given $Q = Vf$

$$\frac{\partial Q}{\partial t} = -\frac{f}{\rho}\frac{\partial p}{\partial x} - 27{,}5\frac{\nu}{f}Q - 0{,}443\frac{k_\varepsilon}{f^{3/2}}Q^2 \quad (15)$$

$$\frac{\partial p}{\partial t} = -\left[\frac{E_L \delta_p E_p}{E_p \delta_p + d_p E_L}\right]\frac{1}{f}\frac{\partial Q}{\partial x}; \quad (16)$$

Consider the changes in the viscosity, density, and modulus of bulk elasticity of a liquid as a function of pressure and at a constant temperature.

The influence of pressure on the dynamic viscosity of the liquid is estimated by the dependence [8]

$$\mu = \mu_0 e^{b(p-p_0)} \quad (17)$$

whereμ, μ_0 – the value of the dynamic viscosity, respectively, at pressures p and p_0 MPa; b – the degree index, the value of which for mineral oils varies in the range of 0.02...0.03 (the lower limit corresponds to high temperatures).

Density changes are determined by the formula [3]

$$\rho = \rho_0 \sqrt[A_a]{A_a p + B_a} \quad (18)$$

whereρ_0 – density value at p_0 and T_0, where $A_a$and B_a– parameters that depend on the type of liquid and its temperature.

The experimental values of the parameters A_a and B_a at 20 ° T ≤ 80 ° for different working fluids are given in [9].

For a number of mineral oils [2, 6], the volume elasticity modulus can be represented by a linear empirical dependence

$$E_L = A_a p + B_a \tag{19}$$

The actual working fluid is a two-phase hydro-air mixture. The air in this mixture can be in a dissolved and undissolved state. Dissolved air practically affects the properties of working fluids [3, 8]. Undisclosed air increases the flexibility of the hydraulic drive and causes a delay in the pressure build-up in the actuators, which has a significant impact on the speed of the entire control system [6]. In the dynamic calculation, it is assumed that the amount of the gas phase in the hydro-air mixture remains constant in the transition process.

The theoretical and experimental study of the volume elastic modulus of a hydro-air mixture is given in the works [6]. In them, depending on the accepted assumptions, different expressions for the isothermal and adiabatic modulus are obtained. In [2], the volume elastic modulus is recommended in the form of:

$$E_L = \frac{a(p_0/p)^{1/n} + (1-a)\sqrt[A_a]{(E_{a0}+A_a p_0)/(E_{a0}+A_a p)}}{a/(np)\cdot(p_0/p)^{1/n} + (1-a)/(E_{a0}+A_a p)} \tag{20}$$

where$a$ – relative initial volume of the gas phase; p_0 and p – initial current fluid pressure; n – polytropy indicator.

Consider a section of a hydraulic drive containing a long pipeline with a capacity at the end of the volume W (Fig.1).

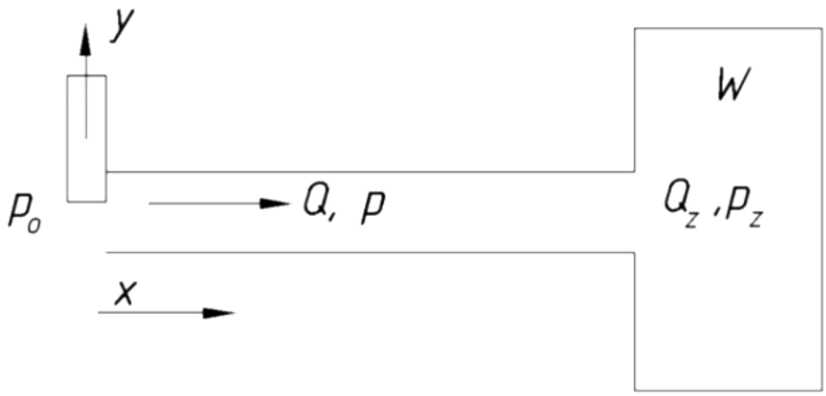

FIGURE 1. Hydraulic drive diagram.

At the beginning of the pipeline, a shut-off spool is installed, in the initial position, blocking the pipeline; pressure in front of the spool p_0 = *const.* As a result of the movement of the spool in the system, non-stationary processes occur. We will write a mathematical model of the considered section of the hydraulic drive.

The motion of the fluid in the pipeline can be described by a system of differential equations (15) – (16).

The flow rate through the shut-off spool is determined by the dependence [9]:

$$Q_S = \mu_e \, f(y)\sqrt{2\,|p_0 - p|/\rho}\, sign(p_0 - p) \qquad (21)$$

where *f(y)* – area of the passage section, μ_e– expense ratio.

The change in pressure in the tank *W* can be described by the equation [6]

$$\frac{\partial p_C}{\partial t} = \frac{E_L}{W} Q_C, \qquad (22)$$

where Q_C – liquid flow rate per container;

The initial and boundary conditions for the system of equations (15) – (22) have the following form.

Initial conditions: when

$$t = 0;\ \ p(x,0) = p_C = 4MPa, \quad Q_S = Q = (x,0) = Q_S = 0, \quad f(y) = 0 \qquad (23)$$

Boundary conditions:

$$\begin{aligned} by\ \ x = 0;\quad & Q(0,t) = Q_S, \\ by\ \ x = 1;\quad & Q_C = Q(l,t), \quad p(l,\ t) = p_C \end{aligned} \qquad (24)$$

Thus, the system of equations (15) – (22), together with the initial and boundary conditions (23) – (24), is a mathematical model of the hydraulic drive section.

For the numerical solution of the proposed mathematical model, the two-layer Lax-Wendroff scheme modified by Burstein is applied [6]. Stability conditions for this scheme $(\Delta t / \Delta x^2) \le 0.5$.

RESULTS

The results of calculations of the mathematical model of the hydraulic drive section shown in Fig. 1 at l=5.2 m, d_p = 0,006 m, p_0 = MPa, d_S = 0,008 m, f_{max} (Y) = 0,0018 in comparison with the experimental data [12] and other methods are shown in Figure 2.

The analysis of the comparative results showed that the average deviation of the results calculated by the formula (in percent)

$$\sigma = \frac{100}{n}\sum_{i=1}^{n}\frac{|f_t(x_i) - f_e(x_i)|}{f_e(x_i)}$$

where $f_t(x_i) - f_e(x_i)$ – theoretical and experimental data at the interpolation points; *n* - the number of points, is: based on a model with focused parameters – 9,1%; by the method of characteristics – 6,47%; according to the proposed model with distributed parameters – 2,8%.

A very important issue in the study of hydraulic drive dynamics is the determination of the limits of the application of models.

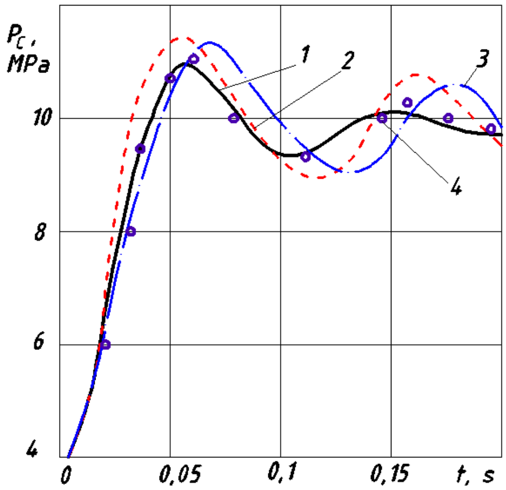

FIGURE 2. Dependence of the pressure change in the tank. 1- by the method with distributed parameters; 2 - by the method with concentrated parameters; 3 - by the method of characteristics; 4 - experimental data.

The article [13] considers the dynamic calculation of a hydraulic brake drive based on a model with distributed parameters and a model with concentrated parameters. Moreover, for the non-stationary tangential stress on the wall, the linear dependence with the velocity is chosen. Based on the analysis of the calculation results and their comparison with experimental data, it is concluded that the model with concentrated parameters gives a general qualitative picture of low-frequency vibrations in the brake drive and can be used in the calculations of hydraulic brake drives with a line length of up to 10 m. The study of the drive according to the proposed model allows us to quantify both low-frequency and high-frequency fluid vibrations and it can be used in the calculations of hydraulic brake drives with a line length of more than 10 m.

Special studies conducted in [6] have shown that for hydraulic systems with a trunk length of less than 5 m, a model with concentrated parameters is acceptable, above 5 m this model gives large errors and it is necessary to apply a model with distributed parameters.

We also consider the question of the limits of applicability of the proposed model with respect to the model with lumped parameters. Studies have shown [14] that the model we proposed can be used in hydraulic drives with a line length of 3 m or more.

CONCLUSION

A mathematical model for the dynamic calculation of a hydraulic drive with distributed parameters is proposed, which makes it possible to study transients in highways with a length of more than 3 m and takes into account the influence of extreme operating conditions.

A comparative analysis of the results of theoretical and experimental studies has shown the adequacy of the proposed models of dynamic calculation to the real processes occurring in hydraulic drives. The average deviation of the results obtained according to the proposed model of dynamic calculation with distributed parameters is 2.8%, which is 3-6% better than other methods.

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