RESULTS OF PROPERTIES OF FIBONACCI NUMBERS

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ABSTRACT

The article presents the properties of Fibonacci numbers with proofs. in addition, some resulting properties are derived from the properties of Fibonacci numbers. the article can be used by students of higher educational institutions and those preparing for the Olympiad in mathematics.

Keywords: *Fibonacci numbers, recurrence relation, golden ratio, major ratio, minor ratio.*

INTRODUCTION

It is known that $u_n = u_{n-1} + u_{n-2} \quad (n \geq 3)$ with $u_1 = u_2 = 1$, the sequence determined by means of recurrent equality is called Fibonacci series, and its terms are called Fibonacci numbers . The phrase Fibonacci numbers can be found in the nineteenth-century works devoted to interesting mathematics by Eduard Lucca. Fibonacci (this word is shortened from the Italian words "filius Bonacci", which means the son of Bonacci) is the nickname of Leonardo Pisanoski, who lived in the city of Pisa in Italy in the 12th and 13th centuries. Bonacci was engaged in trade in Italy and Algeria. Leonardo received his primary education in Algeria. he had learned the Indian positional decimal system and zero from his Arabic teachers.

METHODS

Fibonacci’s "Liber abaci" i.e. "The Book of Abacus" was written in 1202, a manuscript copy of which was preserved in 1228. The book contains almost all the information of his time on arithmetic and algebra. For example, in that book, the Arabic numerals, which are now popular all over the world, are described. On pages 123-124 of the manuscript, the following issue about the breeding of domestic rabbits is described. "A man kept a pair of rabbits for breeding purposes. The nature of the rabbit is such that each pair of rabbits gives birth to another pair of rabbits in one month, and the newly formed pair of rabbits begins to give birth from the second month. will come?" This problem is solved using a table made of Fibonacci numbers, so many problems like this can be solved using Fibonacci numbers and tables, i.e. matrices.

Below we present the properties and methodology of Fibonacci numbers.

Property 1. $u_1 + u_2 + u_3 + \cdots + u_n = u_{n+2} - 1$

Ya’ni $u_3 - u_2 + u_4 - u_3 + \cdots + u_{n+1} - u_n + u_{n+2} - u_{n+1} = u_{n+2} - 1$

Property 2.
$u_1 + u_3 + u_5 + \cdots + u_{2n-1} = u_2 + (u_4 - u_2) + (u_6 - u_4) + \cdots + +(u_{2n} - u_{2n-2}) = u_{2n}$

Property 3.
$u_2 + u_4 + u_6 + \cdots + u_{2n} = (u_1 + u_2 + u_3 + \cdots + u_{2n}) - (u_1 + +u_3 + u_5 + \cdots + u_{2n-1}) = u_{2n+2} - 1 - u_{2n} = u_{2n+1} - 1.$

Property 4. $u_1 - u_2 + u_3 - u_4 + \cdots + (-1)^{n+1}u_n = (-1)^{n+1}u_{n-1} + 1$

Property 5.
${u_1}^2 + {u_2}^2 + \cdots + {u_n}^2 = u_1u_2 + u_2(u_3 - u_1) + u_3(u_4 - u_2) + \cdots + u_n(u_{n+1} - u_{n-1}) = u_nu_{n+1}$

Property 6. ${u_n}^2 - u_{n-1}u_{n+1} = (-1)^{n+1}$

That is, using the method of mathematical induction,

$${u_n}^2 = u_{n-1}u_{n+1} + (-1)^{n+1}$$

$${u_n}^2 + u_nu_{n+1} = u_nu_{n+1} + u_{n-1}u_{n+1} + (-1)^{n+1}$$

$$u_n(u_n + u_{n+1}) = u_{n+1}(u_n + u_{n-1}) + (-1)^{n+1}$$

$$u_n u_{n+2} = u_{n+1}{}^2 + (-1)^{n+1}$$

$$u_{n+1}{}^2 = u_n u_{n+2} + (-1)^n.$$

Property 7. $u_1u_2 + u_2u_3 + u_3u_4 + \cdots + u_{2n-1}u_{2n} = u_{2n}{}^2$

$u_2(u_1 + u_3) + u_4(u_3 + u_5) + \cdots + u_{2n-1}(u_{2n-2} + u_{2n}) =$

$= (u_3 - u_1)(u_1 + u_3) + (u_5 - u_3)(u_5 + u_3) + \cdots$

$\ldots + (u_{2n} - u_{2n-2})(u_{2n} + u_{2n-2}) = u_3{}^2 - u_1{}^2 + u_5{}^2 - u_3{}^2 + u_7{}^2 - u_5{}^2 +$

$+ \cdots + u_{2n}{}^2 - u_{2n-2}{}^2 = u_{2n}{}^2$

Property 8. $u_1u_2 + u_2u_3 + u_3u_4 + \cdots + u_{2n}u_{2n+1} = u_{2n+1}{}^2 - 1$

Property 9. $nu_1 + (n-1)u_2 + (n-2)u_3 + \cdots + 2u_{n-1} + u_n =$

$= (u_1 + u_2 + u_3 + \cdots + u_n) + (u_1 + u_2 + u_3 + \cdots + u_{n-1}) + (u_1 + u_2 + u_3 +$
$+ \cdots + u_{n-2}) + \cdots + u_1 = u_{n+2} - 1 + u_{n+1} - 1 + u_n - 1 + u_{n-1} - 1 + u_1 =$

$= u_{n+4} - (n+3)$

Property 10.

$u_1 + 2u_2 + 3u_3 + \cdots + nu_n = (n+1)(u_1 + u_2 + u_3 + \cdots + +u_n) -$
$(nu_1 + (n-1)u_2 + (n-2)u_3 + \cdots + 2u_{n-1} + u_n) =$

$= (n+1)(u_{n+2} - 1) - (u_{n+4} - (n+3)) = nu_{n+2} - u_{n-3} + 2$

Property 11. $u_n = \sum_{k=1}^{n} C_{n-k-1}^{k}$

Property 12. $u_n = \frac{1}{\sqrt{5}}\left[(\frac{1+\sqrt{5}}{2})^n - (\frac{1-\sqrt{5}}{2})^n\right]$. [1]

RESULTS

$u_{n+2}{}^{\mathbf{2}} - u_{n+1}{}^{\mathbf{2}} = u_n u_{n+3}$

Because

$u_{n+2}{}^{\mathbf{2}} - u_{n+1}{}^{\mathbf{2}} = (u_{n+2} - u_{n+1})(u_{n+2} + u_{n+1}) = u_n u_{n+3}.$

Likewise

$u_{n+m} = u_{n-1}u_m + u_n u_{m+1}$

From this

$u_{n+m} = u_{n+1}u_{m+1} + u_{n-1}u_{m-1}$

CONCLUSION

It can be seen that Fibonacci numbers have an incomparable role in the solution of many natural and life problems. In addition, Fibonacci numbers are also related to the golden section. The golden section of a given section is defined as dividing it into two such parts. here, the ratio of the length of the entire section to the length of the large part and the ratio of the length of the large part to the length of the small part are mutually equal. It is not difficult to determine that the value of this ratio is equal to α_1 . The interesting thing is that

$$\lim_{n\to\infty}\frac{u_{n+1}}{u_n}=\alpha_1$$

will be.

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