SHREDINGER OPERATORI DISKRET SPEKTRINING XOS QIYMATLARI HAQIDA

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[1] ishda $d = 1,2..$ - o‘lchamli $\mathbb{Z}^d$ $\mu > 0$ potensialli ikkita bir xil zarrachali (bozon) sistema panjarada juft-jufti bilan kontakt ta’sirlashuvchi gamiltonianiga mos ikki zarrachali diskret Shredinger operatori $h_\mu(k),\ k \in \mathbb{T}^d, d = 1,2$ qaralgan. Ushbu operator xos qiymati uchun muhim spektr tubidagi yoyilmalar olingan.

[2] ishda ixtiyoriy o‘lchamli panjarada qo‘shni tugunlarda ta’sirlashuvchi ikkita fermionli sistema gamiltonianiga mos $h_\mu(k),\ k \in \mathbb{T}^\nu$ ikki zarrachali diskret Shredinger operatori qaralgan. $h_\mu(k)$ ning zarrachalar ta’sirlashuv energiyasi $\mu > 0$ va sistema kvaziimpul’si $k \in \mathbb{T}^1$ ga bog‘liq muhim spektrdan o‘ngda yoki yagona xos qiymatga ega yoki xos qiymatga ega emasligi isbotlangan.

Ushbu ishda ikki o‘lchamli panjarada qo‘shni tugunlarda ta’sirlashuvchi ikkita fermionli sistema gamiltonianiga mos $h_\mu(k),\ k \in \mathbb{T}^2$ ikki zarrachali diskret Shredinger operatori qaraladi. $h_\mu(k)$ operatorning muhim spektrdan chapda xos qiymatga ega yoki xos qiymatga ega bo‘lmaydigan qiymatlari ajratib ko‘rsatiladi. Sistema kvaziimpulsining ba’zi qiymatlarining aniq ko‘rinishi topilgan.

Faraz qilaylik $\mathbb{T}^2$ - ikki o‘lchamli tor, ya’ni qarama-qarshi tomonlari ayniy $(-\pi,\pi]^2$ kvadrat bo‘lsin. Uni qo‘shish va songa ko‘paytirish amallari $\mathbb{R}^2$ da $(2\pi\mathbb{Z})^2$ modul bo‘yicha qo‘shish va songa ko‘paytirish amallari kabi kiritilgan abel guruhi deb qarash mumkin.

$L_2(\mathbb{T}^2)$ -orqali $\mathbb{T}^2$ da aniqlangan kvadrati bilan integrallanuvchi funksiyalarning gilbert fazosini belgilaymiz.

$L_2^t(\mathbb{T}^2) = \{f \in L_2(\mathbb{T}^2):\ f(-\boldsymbol{q}) = -f(\boldsymbol{q})\}$ – Hilbert fazosida quyidagicha aniqlangan $h_\mu(k),\ k \in \mathbb{T}^2$, operatorni qaraymiz

$$h_\mu(k) = h_0(k) - \mu v.$$

Qo‘zg‘almas $h_0(k)$ operator $\mathrm{E}_k(q)$ funksiyaga ko‘paytirish operatoridir, ya’ni

$$(h_0(k)f)(\boldsymbol{q}) = \mathrm{E}_k(\boldsymbol{q})f(\boldsymbol{q}), \quad f \in L_2^t(\mathbb{T}^2),$$

bunda

$$\mathrm{E}_k(\boldsymbol{q}) = \varepsilon\left(\frac{k}{2}+\boldsymbol{q}\right)+\varepsilon\left(\frac{k}{2}-\boldsymbol{q}\right),$$

$$\varepsilon(\boldsymbol{q}) = \sum_{i=1}^{2}(1-cosq^{(i)}).$$

Ta’sir operatori (qo‘zg‘alish operatori) $v - L_2^t(\mathbb{T}^2)$ Hilbert fazosida quyidagicha aniqlanadi:

$$(vf)(\boldsymbol{q}) = \frac{1}{(2\pi)^2}\sum_{i=1}^{2} sinq^{(i)} \int_{\mathbb{T}^2} sint^{(i)} f(\boldsymbol{t})d\boldsymbol{t}.$$

Ravshanki, $v-$ integral operator bo‘lib, rangi 2 - dan oshmaydi. Muhim spektr turg‘unligi haqidagi Veyl teoremasiga ko‘ra ([4] ga q.) $h_\mu(k)$ operatorning muhim spektri $\sigma_{ess}(h_\mu(k))$ berilgan $\mu \geq 0$ parametrdan bog‘liq emas va h_0 operatorning spektri bilan ustma-ust tushadi. Shunday qilib

$$\sigma_{ess}(h_\mu(k)) = [\varepsilon_{min}(k), \varepsilon_{max}(k)]$$

tenglik o‘rinli, bunda

$$\varepsilon_{min}(k) = min_{q\in\mathbb{T}^2}\, \mathrm{E}_k(\boldsymbol{q}) = 2\sum_{j=1}^{2}\left(1-cos\frac{k^{(j)}}{2}\right) \geq 0,$$

$$\varepsilon_{max}(k) = max_{q\in\mathbb{T}^2}\,\mathrm{E}_k(\boldsymbol{q}) = 2\sum_{j=1}^{2}\left(1 + cos\frac{k^{(j)}}{2}\right) \leq 8$$

$\Pi-$ orqali $(-\pi,\pi]^2$ kubning chegarasini belgilaymiz. Aniqlanishiga ko‘ra

$$\Pi = \left\{\boldsymbol{k} = \left(k^{(1)},\ k^{(2)}\right) \in \mathbb{T}^2:\ birorta\ \ j = 1,2\ \ uchun\ \ k^{(j)} = \pi\right\}$$

Har bir $k \in \mathbb{T}^2\backslash\Pi$ uchun quyidagicha belgilashlarni kiritamiz:

$$\nu^{(i)}(k) = \frac{1}{(2\pi)^2}\int_{\mathbb{T}^2}\frac{sin^2 q^{(i)} d\boldsymbol{q}}{\mathrm{E}_k(\boldsymbol{q}) - \varepsilon_{min}(k)}$$

va

$$\mu_{(i)}(k) = \frac{1}{\nu^{(i)}(k)} > 0.$$

Aytish joizki, har bir $\boldsymbol{k} \in \mathbb{T}^2\backslash\Pi$ uchun

$$\mathrm{E}_k(\boldsymbol{q}) = 2\sum_{j=1}^{2}(1 - cos\frac{k^{(j)}}{2}cosq^{(j)})$$

funksiya $q = (0,0)$ nuqtada aynimagan minimumga ega bo‘lganligi uchun yuqoridagi integral chekli.

$\Pi_{k_1} -$ orqali quyidagi to‘plamni belgilaylik:

$\Pi_{k_1} = \{\boldsymbol{k} \in \mathbb{T}^2:\ \ k_2 = \pi\}$

Teorema. $\boldsymbol{k_1} = (\boldsymbol{k}, \boldsymbol{\pi})$ bo‘lsin, ya’ni $\boldsymbol{k} \in \Pi_{k_1}$. U holda $0 < \mu < 2\cos\frac{k_1}{2}$ bo‘lsa,

$h_\mu(k_1)$ operator muhim spektrdan chapda yagona xos qiymatga ega va u $\mu > 2\cos\frac{k_1}{2}$ bo‘lsa, muhim spektrdan chapda ikkita xos qiymati mavjud hamda ushbu xos qiymatlar quyidagi ko‘rinishga ega:

$$z = 4 - \sqrt{4cos^2\frac{k^{(2)}}{2} + \frac{\mu^2}{4}},$$

$$z = 4 - \left(\frac{2cos\frac{k_1}{2}}{\mu} + \frac{\mu}{2}\right).$$

Endi ishimizni ikkinchi qismiga o‘tamiz.

Har bir $k \in \mathbb{T}^2 \setminus \Pi$ uchun quyidagi to‘plamni kiritamiz:

$M_>(k) = \{\mu \in \mathbb{R}_+ : \ \mu > \mu_i(k), \ \ i = 1,2\}$

Teorema. $\mu \in M_>(k), \ i = 1, 2, \ \ k \in \mathbb{T}^2$ bo‘lsin. U holda $h_\mu(k)$ operator muhim spektrdan chapda ikkita xos qiymatga ega.

Eslatma. $k_1 = (\pi; 0)$ bo‘lsin, ya’ni $k_1 \in \Pi$. U holda $\mu_{(1)}(k_1) = 0 \ va \ \mu_{(2)}(k_1) = 2$ tenglik o‘rinli.

Teorema. $k = k_1$ bo‘lsin. U holda agar $0 < \mu < 2$ bo‘lsa, $h_\mu(k_1)$ operator muhim spektrdan chapda yagona xos qiymatga ega va $\mu > 2$ bo‘lsa ikkita xos qiymati mavjud hamda ushbu xos qiymatlar quyidagi ko‘rinishga ega:

$$z^{(1)}(\mu, k_1) = 4 - \sqrt{4 + \frac{1}{4}\mu^2},$$

$$z^{(2)}(\mu, k_1) = 4 - \left(\frac{\mu}{2} + \frac{2}{\mu}\right).$$

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