A NOVEL PROBABILISTIC METHOD FOR ENERGY LOSSESTIMATION USING MINIMAL LINE CURRENT INFORMATION

X.Z.Nazirova., O.Z.Nazirova., Sh.A.Abdunazarov..

Tashkent State Technical University

Annotation. *Despite it is essential to distribution networkeconomics, computing energy loss for most major networks is still a tough task due to the absence of full monitoring.Assume the line current follows a normal distribution, the sum of its square is a linear combination of independent chi-square variables, which follows a generalized noncentral chi-square distribution. Based on this finding, we develop anew probability-based analytical method to estimate distribution network energy losses efficiently. The proposedanalytical method requires merely the knowledge of mean and variance of line current as well as the line resistance while provides a closed-form formula of the probability characteristics of energy loss. The method is demonstrated on a three-feeder radial network.*

Keywords: *Energy losses, distribution system, normaldistribution, probability.*

Computing energy loss, especially technical loss isimportant for the operation and planning of distribution networks. However, an accurate energy loss computation relies on detailed information about distribution system. This information (e.g. real-time load curve, accurate line resistance,and low-voltage network structure) is often unavailable inexisting distribution systems, because of the prohibitively highcost associated with monitoring millions or more distribution networks. Therefore, a feasible alternative approach is to estimate the energy loss based on the rule of thumb. For example, a prevailing energy loss estimation approach is called *loss factor*, which determines the energy losses by a product of peak load and a suitable factor. However, there is no evidence to support the direct relationship between maximum demand and energy losses [1], the *loss factor* is determined subjectively.More often than not, the factor is a guess.

The lack of a statistical explanation for the principle and thelow accuracy of the estimation of distribution network energy losses motivate researchers to explore novel loss estimation methods [2]. However, the main barrier of energy loss estimation sits in the square of line currents. For an accurate estimate of energy losses on a distribution asset within period T, it is necessary to calculate the integral [3],

$$\Delta W = r\int_{0}^{T}[I(t)]^2\,dt \qquad (1)$$

where ΔW, r, and I denote the energy loss, resistance, and current of the element, respectively. Since the current curve at each distribution system element is not usually measured, the sum of its square is also unavailable.

To this end, this letter proposes a novel probability theory- based approach for estimating the energy loss of distribution systems. In this letter, the line current of each feeder is assumed to have a normal distribution, though mathematical derivation, we showed the energy loss, therefore, has a generalized noncentral Chi-square distribution. The distribution system planner and operator can quickly derive the expectation value and the moment generating function of energy loss without requiring the prohibitively expensive knowledge of load curves. Furthermore, the proposed approach will provide a theoretical basis for energy loss estimation in vast inadequate metering distribution networks, thus better reflecting the economics in decision making of distribution system planning.

Suppose the line current array $\boldsymbol{I} = (I1, \ldots, Ip)$ of different feeders can be presented as a random vector with a multi- normal distribution having expectation vector $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}$. For a practical radial distribution network, the covariance matrix $\boldsymbol{\Sigma}$ is nonsingular due to the diversity of load curves. The energy loss can be calculated as,

$$\Delta W(\boldsymbol{I}) = \Delta t \cdot \boldsymbol{I}^T \boldsymbol{R} \boldsymbol{I} = \boldsymbol{I}^T \boldsymbol{A} \boldsymbol{I} = \sum_{j \le p} a_j I_j^2 \qquad (2)$$

where ΔW denotes the total energy loss; Δt denotes the time interval; $\boldsymbol{R}$ is the matrix of line resistance rj; aj is a parameter that equals $rj\times\Delta t$; $\boldsymbol{A}$ is a diagonal matrix of aj. If the time interval Δt and line resistance rj are fixed, the matrix $\boldsymbol{A}$ will be a constant matrix.

In order to facilitate the analysis, we need to transform the line current $\boldsymbol{I}$ to a standard normal variable. Let $\boldsymbol{Y} = \boldsymbol{\Sigma}^{-1/2}\boldsymbol{I}$, the line current variable $\boldsymbol{I}$ is now converted to a new variable $\boldsymbol{Z} = (\boldsymbol{Y} - \boldsymbol{\Sigma}^{-1/2}\boldsymbol{\mu})$ with zero expectation and identity variance matrix. Then, the total energy loss is reformulated as,

$$\Delta W(\boldsymbol{I}) = \boldsymbol{I}^T \boldsymbol{A}\boldsymbol{I} = (\boldsymbol{Z} + \boldsymbol{\Sigma}^{-1/2}\boldsymbol{\mu})^T \boldsymbol{\Sigma}^{1/2}\boldsymbol{A}\boldsymbol{\Sigma}^{1/2}(\boldsymbol{Z} + \boldsymbol{\Sigma}^{-1/2}\boldsymbol{\mu}) \quad (3)$$

For the middle term of equation (3), the eigen decomposition is enforced to further simplify the equation.

$$\boldsymbol{\Sigma}^{1/2}\boldsymbol{A}\boldsymbol{\Sigma}^{1/2} = \boldsymbol{P}^T \boldsymbol{\Lambda}\boldsymbol{P} \quad (4)$$

where $\boldsymbol{\Lambda}$ is a diagonal for eigenvalues $\lambda 1, \ldots, \lambda p$ of $\boldsymbol{\Sigma}^{1/2}\boldsymbol{A}\boldsymbol{\Sigma}^{1/2}$ $\boldsymbol{P}$ is an $p \times p$ orthogonal matrix ($\boldsymbol{P}\boldsymbol{P}^T = \boldsymbol{P}^T\boldsymbol{P} = \boldsymbol{E}$, $\boldsymbol{E}$ is an identity matrix). Then the total energy loss is represented as,

$$\Delta W(\boldsymbol{I}) = \boldsymbol{I}^T \boldsymbol{A}\boldsymbol{I} = (\boldsymbol{Z} + \boldsymbol{\Sigma}^{-1/2}\boldsymbol{\mu})^T \boldsymbol{\Sigma}^{1/2}\boldsymbol{A}\boldsymbol{\Sigma}^{1/2}(\boldsymbol{Z} + \boldsymbol{\Sigma}^{-1/2}\boldsymbol{\mu}) = (\boldsymbol{Z} + \boldsymbol{\Sigma}^{-1/2}\boldsymbol{\mu})^T \boldsymbol{P}^T \boldsymbol{\Lambda}(\boldsymbol{Z} + \boldsymbol{\Sigma}^{-1/2}\boldsymbol{\mu})\boldsymbol{P} = (\boldsymbol{P}\boldsymbol{Z} + \boldsymbol{P}\boldsymbol{\Sigma}^{-1/2}\boldsymbol{\mu})^T \boldsymbol{\Lambda}(\boldsymbol{P}\boldsymbol{Z} + \boldsymbol{P}\boldsymbol{\Sigma}^{-1/2}\boldsymbol{\mu}) \quad (5)$$

Define new variables $\boldsymbol{U} = \boldsymbol{P}\boldsymbol{Z}$ and $\boldsymbol{b} = \boldsymbol{P}\boldsymbol{\Sigma}^{-1/2}\boldsymbol{\mu}$ to simplify the notations, the energy loss is finally written as,

$$\Delta W(\boldsymbol{I}) = \boldsymbol{I}^T \boldsymbol{A}\boldsymbol{I} = (\boldsymbol{U} + \boldsymbol{b})^T \boldsymbol{\Lambda}(\boldsymbol{U} + \boldsymbol{b}) = \sum_{j=1}^{p} \lambda_j (U_j + b_j)^2 \quad (6)$$

Note, $\boldsymbol{U}$ is actually a standard normal distributed variable with zero expectation and identity variance matrix, $\boldsymbol{b}$

is a constant array.

The above transformations imply that the energy loss $\Delta W(\boldsymbol{I})$ is a linear combination of independent central chi-square variables when $\boldsymbol{\mu}=\boldsymbol{0}$ and of noncentral chi-square variables when $\boldsymbol{\mu}\neq\boldsymbol{0}$. For actual distribution networks, the line current has positive expectation values ($\boldsymbol{\mu} \neq \mathbf{0}$), in this case, the corresponding probabilistic distribution for energy loss $\Delta W(\boldsymbol{I})$ is called generalized noncentral chi-square distribution, denote as

$$\Delta W(\boldsymbol{I}) \sim G\chi^2 (\delta^2) \qquad (7)$$

where p denotes the degrees of freedom; δ^2 denotes the non- centrality parameter, $\delta^2 = \boldsymbol{b}^T \boldsymbol{\Lambda} \boldsymbol{b}$.

The work in [4] gives an integral form of probabilistic density function (PDF) and cumulative distribution function (CDF) of the generalized noncentral chi-square distribution. The moment generating function (MGF) is given in [5], as

$$\mu_{\Delta W} = E[\Delta W(\boldsymbol{I})] = tr(\boldsymbol{A\Sigma}) + \boldsymbol{\mu}^T \boldsymbol{A\mu} \qquad (8)$$

$$\sigma^2_{\Delta W} = Var[\Delta W(\boldsymbol{I})] = E[\Delta W(\boldsymbol{I})]^2 = 2tr(\boldsymbol{A\Sigma})^2 + 4\boldsymbol{\mu}^T \boldsymbol{A\Sigma A\mu} \qquad (9)$$

However, apart from certain simple cases (e.g., $\boldsymbol{A}=\boldsymbol{\Sigma}=\boldsymbol{E}$), there's no closed exact expression of PDF and CDF for generalized noncentral chi-square distribution [6]. Therefore, approximation approaches like saddle-point approximation [7] and moment-matching [8] can be applied to obtain its probability value.

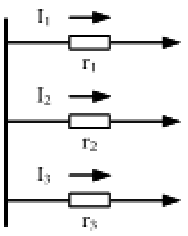
Note, the expectation value of $\Delta W(\boldsymbol{I})$ represent the expected energy loss in the time interval Δt. Since the resistance does not change with time, multiply the expectation value by the ratio of a specified time interval, one can obtain the expected energy loss immediately.

A significant advantage of the proposed method is its convenience that one can get the expected energy loss immediately from equation (9) with a small number of parameters like mean and variance of line current and the line resistance. Since there is no independent assumption about line currents, this method can easily extend to multi-feeder or low voltage three-phase distribution network scenarios, even if the currents are positively correlated.

Furthermore, different from the experience-based energy loss estimation method that only focuses on the expectation of energy loss, the proposed method is able to provide more probabilistic information about energy loss, like moments and probability distributions. This additional probabilistic information that provided by the proposed method will enable the distribution system planner to apply advanced risk assessment method (e.g., conditional value at risk) in the distribution network planning scheme, and thus improve the economy of distribution network.

The demonstration of the proposed distribution network

Load 1 $r_1=0.01\Omega$



Load 2 $r_2=0.02\Omega$

Load 3 $r_3=0.03\Omega$

Fig. 1. Simple three-feeder circuit for analysis.

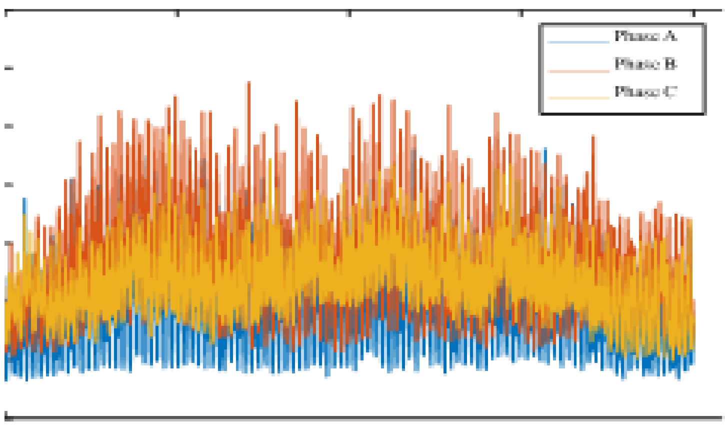
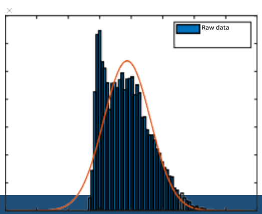
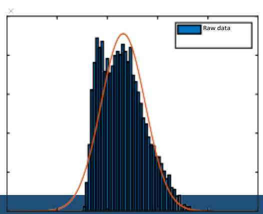
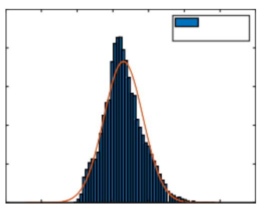


Fig. 2. Load curves for analysis.

a) b) c)

Fig. 3. Histograms and fitted normal distributions for each load curve. Subpicture (a), (b), (c) represents the corresponding three phases.

energy loss estimation method is on an ideal three-feeder radial network shown in Fig. 1. The load data shown in Fig. 2 come from an unbalanced three-phase low-voltage distribution network in the UK. The data set contains a measurement of line current for 100 days with a time interval of 10 min (144 data points per day for each phase). As a comparison, the practical energy loss is calculated based on the load curve and line resistance. The numerical test is performed on a computer with Intel Xeon E5-2650v4 (2.20 GHz) CPU and 16 GB RAM. The program is implemented using Matlab 2018b.

The histograms of raw data and their fitted marginal normal distributions are shown in Fig. 3. The mean and variance value for each marginal normal distribution are listed in Table I. The

TABLE I

MEAN AND VARIANCE OF FITTED NORMAL DISTRIBUTIONS

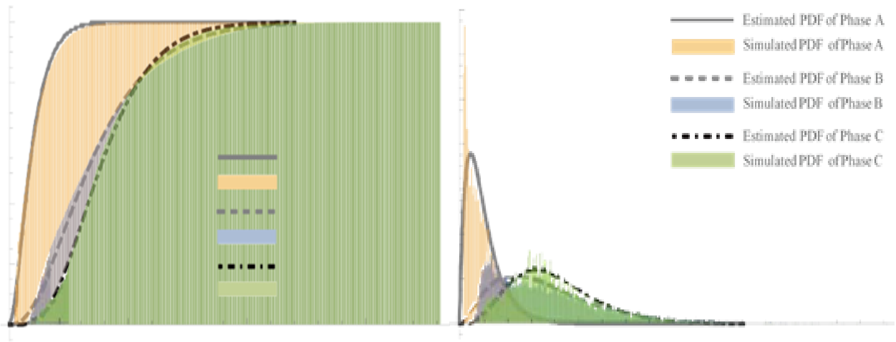
	Phase A	Phase B	Phase C
Mean (A)	187.05	262.9	229.56
Variance (A^2)b	74.16	87.63	54.64

covariance matrix for the multivariable normal distribution is given in equation (11).

$$COV = \begin{bmatrix} 5.46 & 7.68 & 3.43 \\ 2.92 & 3.42 & 2.99 \end{bmatrix} \times 10^3$$

(10)

From Fig. 2 and Table I, we can see phases A, B, and C are heavily unbalanced, where phase B burdens the “heaviest” load and phase A has the “lightest” load. From the aspect of the probability distribution, phase C can be fitted well with normal distribution while the other two phases are not symmetrical and possess obvious fitting errors. Furthermore, the covariance



a) b)

Fig. 4. PDF and CDF of estimated three-phase energy loss. (a) PDF, (b) CDF.

calculated by equation (9), where the time interval Δt is set to 600 seconds and the result of μ is multiplied by 14400 to estimate the total energy loss in the whole measurement time. Computing results of the proposed methods and related estimating error (abs(A-B)/A×100, where A denotes actual results, B denotes estimation results) are listed in Table II. To compare with the other energy loss estimation methods, results from the loss coefficient method proposed in [1] and the loss factor method are also provided. To be specific, the loss factor is carried out by equation (11), and the loss coefficient is carried out by equation (12).

$$loss\ factor \ \square \ \underline{\ \Sigma\ load\ \ } \ (11)$$

$$N \ \square \ load^{2}_{ma}$$

where N is the number of load intervals; load2 i is the load value for load interval i; load2 max is the maximum load demand.

TABLE III

PARAMETERS OF ESTIMATED GAMMA DISTRIBUTION

	Phase A	Phase B	Phase C
Shape parameter	1.97	2.63	4.79
Scale parameter	492.14	1400.81	836.76

$$\textit{loss coefficient} = CV^2 + 1 \qquad (12)$$

where CV is the coefficient of variation of load curve, which is computed by the ratio between the standard deviation and mean of line currents.

The results in Table II give an intuitive comparison between proposed method and the existing methods. As is illustrated in Table II, the proposed method is as accurate as the *loss coefficient* method proposed in [1] and far more accurate than the traditional *loss factor* method. Since the estimated distributions of phase A and phase B are a little bit deviate from the normal distribution, the estimating errors of phase A and phase B are higher than that of phase C.

Furthermore, different from the deterministic methods (e.g., *loss coefficient* and *loss factor*) that only focus on the expectation value of the energy loss, the proposed method is able to provide more information about energy loss probability distribution. Although the accurate probability distribution functions cannot be easily derived, they can still be approximated by the moment generating function. To illustrate this feature, a Gamma distribution based moment-matching method [8] is applied to approximation the PDF and CDF of the energy loss estimation. The shape and scale parameters of the estimated Gamm distribution are given in Table III. Meanwhile, a Monte Carlo simulation is run to offer a reference for probability distribution estimation. Finally, the estimated and simulated energy loss probability distribution is shown in Fig. 4.

In Fig.4, the Monte Carlo simulated energy loss distributions are given by histograms and displayed by different colored areas. The estimated distributions are plotted by lines. As is presented in Fig.4 the proposed method is able to provide a good fit for the energy loss distribution. Through this estimated distribution, distribution network planners are able to figure out the estimation error of the energy loss and choose a confidence interval to enhance the robustness of energy loss estimation according to the probability distribution. All in all, like the probability forecasting method in the load forecast research, the proposed method is not only able to provide a reliable energy loss estimation result but also offer useful probability information of energy loss, which values more in the highly randomized modern distribution network.

This letter presents an explicit probabilistic formula for the energy loss of distribution networks. Based on the proposed formula, this paper presents a novel probabilistic energy loss estimation method in distribution networks. In the proposed approach, only the information of line resistance and the mean and variance value of the line current is needed to estimate the energy loss and its moment generating function. This feature makes it possible to quickly and accurately estimate the energy loss in the incipient or inadequate metering distribution systems. By applying this method, distribution network planners can get the valuable probability information of energy loss, and thus, make a better network expansion decision after carefully assessing the economic risk. Further studies can employ the Gaussian mixed model and thus improve the accuracy of the method.

LITERATURE

1. Losses Estimation in Power Distribution Systems," IEEE Transactions on Power Systems, vol. 27, no. 4, pp. 1879-1887, Nov. 2012.

2. L. Fang, K. Ma, R. Li, Z. Wang, and H. Shi, "A Statistical Approach to Estimate Imbalance-Induced Energy Losses for Data-Scarce Low Voltage Networks," IEEE Transactions on Power Systems, vol. 34, no. 4, pp. 2825-2835, Jul. 2019.

3. O. M. Mikic, "Variance-Based Energy Loss Computation in Low Voltage Distribution Networks," IEEE Transactions on Power Systems, vol. 22, no. 1, pp. 179-187, Fed, 2007.

4. T. Cacoullos and M. Koutras, "Quadratic forms in spherical random variables: Generalized noncentral x2 distribution," Naval Research Logistics Quarterly, vol. 31, no. 3, pp. 447-461, 1984.

5. A. Mathai and S. Provost, "Quadratic Forms in Random Variables: Theory and Applications," vol. 87, 1992.

6. P. Ramírez-Espinosa, D. Morales-Jimenez, J. A. Cortés, J. F. Paris, and E. Martos-Naya, "New Approximation to Distribution of Positive RVs Applied to Gaussian Quadratic Forms," IEEE Signal Processing Letters, vol. 26, no. 6, pp. 923-927, Jun. 2019.

7. R. B. Davies, "Algorithm AS 155: The Distribution of a Linear Combination of χ2 Random Variables," Journal of the Royal Statistical Society. Series C (Applied Statistics), vol. 29, no. 3, pp. 323-333, 1980.