TUPROQNING MUZLASH MUAMMOSINI RAQAMLI MODELLASHTIRISH. TO‘R TUGUNIDA FRONTNI USHLASH USULINING HISOBLASH SXEMASI

Jo‘rayeva D.Sh.
National University of Uzbekistan , Tashkent
e-mail: dildorajurayeva2021@gmail.com

Yarmetova D.I.
National University of Uzbekistan , Tashkent
e-mail: saidovadilafruz1996@mail.ru

Ubaydullayeva G.O.
National University of Uzbekistan , Tashkent
e-mail: ubaydullayevagulnora@mail.ru

ABSTRACT

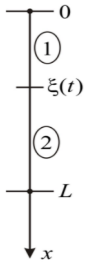
The article presents a mathematical model of the process of freezing wet soil and evaluates the approximation of this model.In modeling the freezing process of the soil, we included and used various thermal and physical properties.It should be noted that they are the same for the thawed and frozen ground zones. That is, the material properties of solid and liquid phases were considered constant.The scheme of the heat exchange process in the soil is described by the method of catching the front (Lovli front) in the mesh node of the freezing of wet soil.The first phase is the molten phase, and the second phase is the frozen phase.The grid step in x, where the grid is built, is equal to the constant h. It was mentioned that the grid step in time is not constant.

***Key words**: soil, phase, freezing temperature, thermal conductivity*

1. Introduction

Nam tuproq (1-rasm) erigan holatda va boshlang‘ich doimiy harorat T_0 ga ega. Dastlabki vaqtda yer yuzasida birdaniga bir oz harorat T_c o‘rnatilgan, T_c T_f (muzlash harorati)dan pastdir. Natijada, o‘zgaruvchanning muzlatilgan qatlami hosil bo‘ladi.(qalinligi $\xi = s(t)$) Uning pastki harakatlanuvchi chegarasida har doim muzlash harorati T_f mavjud. Bu chegarada bir agregatsiya holatidan ikkinchi holatga o‘tish sodir bo‘ladi, bu Q(J/kg) o‘tish issiqligini talab qiladi. Shunday qilib, eritilgan zonaning yuqori chegarasi $x = \xi$ doimiy muzlash haroratiga ega, x=L pastki chegarasi esa tuproqning katta chuqurlikdagi doimiy haroratga ega. Muzlatilgan va eritilgan zonalarning transport koeffitsientlari har xil.[2]

Tuproqdagi issiqlik almashinuvi faqat issiqlik o‘tkazuvchanligi tufayli sodir bo‘ladi deb taxmin qilinadi.



1-rasm (Yechim maydoni)

Yuqorida aytilganlarga asoslanib, matematik bayonot vazifalar quyidagicha ko‘rinadi:

$$\begin{cases} \frac{\partial T_1}{\partial t} = \alpha_1 \frac{\partial^2 T_1}{\partial x^2}, 0 < x < \xi(t), t > 0 \\ \frac{\partial T_2}{\partial t} = \alpha_2 \frac{\partial^2 T_2}{\partial x^2}, \xi(t) < x < L, t > 0 \end{cases} \quad (1)$$

$$t = 0 : T(x) = T0 ,0 < x < L;$$

$$x = 0 : T(x) = T_c ,t > 0; x = L : \frac{\partial T}{\partial x} = 0 ,t > 0; \quad (2)$$

$$x = \xi(t) \begin{cases} T_1 = T_2 = T_f \\ \lambda_1 \frac{\partial T_1}{\partial t} - \lambda_2 \frac{\partial T_2}{\partial t} = Q\rho\omega(1-m)\frac{d\xi}{dt} \end{cases}$$

Bu yerda $\alpha_1 = \frac{\rho_1 c_1}{\lambda_1}$, $\alpha_2 = \frac{\rho_2 c_2}{\lambda_2}$

$$\rho_1 c_1 = (1-m) c_t \rho_t + m c_m \rho_m$$

$$\rho_2 c_2 = (1-m) c_t \rho_t + m c_s \rho_s$$

$$\lambda_1 = (1-m)\lambda_t + m\lambda_m$$

$$\lambda_2 = (1-m)\lambda_t + m\lambda_s$$

$\rho = \frac{\rho_1 + \rho_2}{2}$ – bu yerda qattiq va suyuq fazalarning zichligining o‘rta arifmetkidir.

Muammoni shakllantirishda ishtirok etadigan doimiy haroratlar: $T_c < T_f < T_0$

Tuzilgan muammo fazoviy to‘rning tugunida frontni ushlash usuli bilan hal qilinadi.

Keling, x o‘zgaruvchiga nisbatan yagona to‘rni kiritamiz

$$x_i = (i-h)h, h = \frac{L}{N-1}, i = \overline{1,N}\ x_1 = 0, x_N = L \quad (3)$$

va vaqtga nisbatan ham to‘rni kiritamiz

$$t_{n+1} = t_n + \tau_{n+1},\quad n = \overline{0,N-1}, t_0 = 0, t_M = t_{tugatish}, \tau_{n+1} > 0 \quad (4)$$

Keling, τ_{n+1} qadamini shunday tanlaylikki, bu vaqt ichida fazaviy o‘tish chegarasi fazoviy panjara bo‘ylab to‘liq bir qadam harakat qiladi.

$$\frac{\partial \xi}{\partial t} \approx \frac{h}{\tau_{n+1}} \quad (5)$$

Keling, asl muammoni ikkita kichik vazifaga ajratamiz (erigan va muzlatilgan tuproq zonalarida). Har bir sxemani hal qilish uchun biz progonka usulidan foydalanamiz.

Tuproqning muzlatilgan qismida issiqlik o‘tkazuvchanligi muammosi (1) - (2) ayirmali sxemasi bilan yaqinlashtiriladi.

$$\frac{T_{1,j}^{n+1} - T_{1,j}^{n}}{\tau_{n+1}} = \frac{T_{1,j+1}^{n} - 2T_{1,j}^{n+1} + T_{1,j-1}^{n+1}}{h^2}, i = \overline{2, i^* - 1};$$

$$T_1|_{i=1} = T_c;$$

$$T_1|_{i^*=1} = T_f;$$

(6)

bu yerda i=i*-fazalar o‘tish chegarasi.

Olingan tizimni eng umumiy shaklga keltirish mumkin:

$$A_i T_{1,i+1}^{n+1} - B_i T_{1,i}^{n+1} + C_i T_{1,i-1}^{n+1} = F_i \quad (7)$$

$$A_i = C_i = \frac{a_1}{h^2}, B_i = \frac{2 * a_1}{h^2} + \frac{1}{\tau_{n+1}}, F_i = -\frac{T_{1,i}^{n}}{\tau_{n+1}}$$

Progonka koeffitsientlari (9) formulalar bo‘yicha topiladi.

Faraz qilaylik, α_i va β_i $(i = \overline{1, N-1})$ sonlar to‘plami borki, (7) tenglamani (6) ga aylantiradi.

$$T_i^{n+1} = \alpha_i T_{i+1}^{n+1} + \beta_i \quad (8)$$

(3.1.6) ga nisbatan indeksni bittaga kamaytiramiz va natijada olingan $T_{i-1}^{n+1} = \alpha_{i-1} T_i^{n+1} + \beta_{i-1}$ ifodasini (3.1.5) tenglamaga almashtiramiz:

$$A_i T_{i+1}^{n+1} - B_i T_i^{n+1} + C_i \alpha_{i-1} T_i^{n+1} + C_i \beta_{i-1} = F_i \quad (9)$$

(8) tenglamadan biz (10) ni olamiz.

$$T_i^{n+1} = \frac{A_i}{B_i - C_i \alpha_{i-1}} T_{i+1}^{n+1} + \frac{C_i \beta_{i-1} - F_i}{B_i - C_i \alpha_{i-1}} \quad (10)$$

Oxirgi tenglik (8) shaklga ega va agar barcha i=2, 3…, N-1 uchun (11) munosabatlari bo‘lsa, unga to‘liq mos keladi.

$$\alpha_i = \frac{A_i}{B_i - C_i\alpha_{i-1}}$$
$$\beta_i = \frac{C_i\beta_{i-1} - F_i}{B_i - C_i\alpha_{i-1}} \quad (11)$$

(3.1.9) orqali α_i va β_i ni aniqlash uchun chap chegara shartidan topilgan α_1 va β_1 ni bilish kerak. Bundan tashqari, (8) formulalarga muvofiq $T_{N-1}^{n+1}, T_{N-2}^{n+1}, ..., T_2^{n+1}$ ketma-ket topiladi, T_N^{n+1} to‘g‘ri chegara shartidan topilgan bo‘lsa.Shunday qilib, progonka usuli deb ataladigan tavsiflangan usul bilan (7) ko‘rinishdagi tenglamalarni hal qilish uchta formuladan foydalangan holda hisob-kitoblarga tushiriladi:(11) formulalar bo‘yicha $(i = \overline{2, N-1})$ (to‘g‘ri progonka) α_i va β_i progonka koeffitsientlarini topish.keyin i=N-1, N-2, …, 2 da (8) formula bo‘yicha noma’lum T_i^{n+1} ni olish (teskari progonka). Progonka usulini muvaffaqiyatli qo‘llash uchun quyidagilar zarur:Shunday qilib, hisob-kitoblar jarayonida nolga bo‘linadigan holatlar mavjud emas va tizimlarning katta o‘lchamlari bilan yaxlitlash xatolarining tez o‘sishi bo‘lmasligi kerak. Agar supurish koeffitsientlarining (11) maxrajlari yo‘qolmasa va barcha $(i = \overline{1, N-1})$ uchun $|\alpha_i| < 1$ bo‘lsa progonka metodi to‘g‘ri ishlayotgan bo‘ladi.[2] da yetarli shartlar keltiruvchi teorema isbotlangan

Tenglamalarning to‘g‘riligi va barqarorligi (7):

$$|B_i| > |A_i| + |C_i| \ \ \forall i = \overline{2, N-1} \text{ va } |\alpha_1| < 1 \Rightarrow |\alpha_i| < 1 \quad (12)$$

Tizimga (6) qaytib, biz progonkani aniqlaymiz.Koeffitsientlar va natijada olingan tizimni yechish uchun to‘liq algoritmni qayta yaratamiz.

Chunki x =0 T=T_c,

$$T_1^{n+1} = \alpha_1 T_2^{n+1} + \beta_1 = T_c$$
$$\alpha_1 = 0, \beta_1 = T_c$$

Tuproqning ikkinchi qismidagi ayirmali sxemasini ko‘rib chiqing.(1-2) sistemaning ikkinchi tenglamasini diskretlash uchun biz to‘rt nuqtali yashirin sxemadan ham foydalanamiz.

$$\frac{\partial T_2}{\partial t} = \alpha_2 \frac{\partial^2 T_2}{\partial x^2}, \xi(t) < x < L, t > 0$$

$$\frac{T_{2,j}^{n+1}-T_{2,j}^{n}}{\tau_{n+1}}=\frac{T_{2,j+1}^{n}-2T_{2,j}^{n+1}+T_{2,j-1}^{n+1}}{h^2}, i=i^*+1, N-1;$$
$$T_2|_{i=i^*}=T_f; \quad (13)$$
$$\frac{\partial T_2}{\partial x}|_{i=i^*}=0.$$

Progonka koeffitsientlari (11) formulalar bo‘yicha topiladi.Bundan tashqari, noma’lum harorat maydoni (8) ifoda bilan aniqlanadi.

Chegaraviy shartni diskretlashtiramiz $x=\xi(t)$

$$\lambda_1\frac{\partial T_1}{\partial t}-\lambda_2\frac{\partial T_2}{\partial t}=Q\rho\omega(1-m)\frac{d\xi}{dt}.$$

Bu chegaraviy shart vaqt bosqichini har safar aniqlash uchun zarur.O(h) xatosi bilan diskretlashtiramiz.

$$\lambda_1\frac{\partial T_1}{\partial x}\bigg|-\lambda_2\frac{\partial T_2}{\partial x}\bigg|=Q(1-m)\rho w\frac{d\xi}{dt};$$

$$\lambda_1\left(\frac{T_{1,i^*}-T_{1,i^*-1}}{h}\right)-\left(\frac{T_{2,i^*}-T_{2,i^*-1}}{h}\right)=Q\rho(1-m)\omega\frac{h}{\tau_{n+1}}.$$

Keyinchalik, biz indekslarni o‘tkazib yuboramiz, $i<i*$ da-1-qism va $i>i*$ da-2-qism ekanligiga asoslanib, tuproqning ko‘rib chiqilayotgan qismini tavsiflaydi.

$$\lambda_1\left(\frac{T_f-T_{i^*-1}}{h}\right)-\lambda_2\left(\frac{T_{i^*+1}-T_f}{h}\right)=Q\rho(1-m)\omega\frac{h}{\tau_{n+1}}.$$

Va natijada

$$\tau_{n+1}=\frac{Q\rho(1-m)h^2}{\lambda_1(T_f-T_{i^*-1})-\lambda_2(T_{i^*+1}-T_f)}.$$

Vaqt qadami haroratga bog‘liqligini ko‘ramiz.Keyinchalik, harorat maydonini aniqlash uchun oddiy iteratsiya usulini qo‘llash kerak.Yuqorida aytib o‘tilgan asosiy g‘oya.

Nochiziqli chegaraviy shartni (2) O(h^2) xatosi bilan diskretlashtiramiz. [3] [1]

Teylor qatorida $x=\xi$ nuqtaga yaqin joylashgan T(x) funksiyani h ga nisbatan ikkinchi tartibli hadlargacha kengaytiramiz:

$$T_{i^*+1}^{n+1}=T_{i^*}^{n+1}+h\cdot\frac{\partial T_2}{\partial x}\bigg|_{x=\xi}^{n+1}+\frac{h^2}{2}\cdot\frac{\partial^2 T_2}{\partial x^2}\bigg|_{x=\xi}^{n+1};$$

$$T_{i^*-1}^{n+1}=T_{i^*}^{n+1}+h\cdot\frac{\partial T_1}{\partial x}\bigg|_{x=\xi}^{n+1}+\frac{h^2}{2}\cdot\frac{\partial^2 T_1}{\partial x^2}\bigg|_{x=\xi}^{n+1}.$$

(1-2) munosabatlaridan foydalanib, biz quyidagilarni olamiz:

$$\left.\frac{\partial^2 T_1}{\partial x^2}\right|_{x=\xi}^{n+1} = \frac{1}{a_1}\left.\frac{\partial T_1}{\partial t}\right|_{x=\xi}^{n+1} = \frac{T_{i^*}^{n+1} - T_{i^*}^{n}}{a_1 \tau_{n+1}};$$

$$\left.\frac{\partial^2 T_2}{\partial x^2}\right|_{x=\xi}^{n+1} = \frac{1}{a_2}\left.\frac{\partial T_2}{\partial t}\right|_{x=\xi}^{n+1} = \frac{T_{i^*}^{n+1} - T_{i^*}^{n}}{a_2 \tau_{n+1}}.$$

Keyin

$$\lambda_1 \left.\frac{\partial T_1}{\partial x}\right|_{x=\xi}^{n+1} = \frac{\lambda_1}{h}\left(T_{i^*}^{n+1} - T_{i^*-1}^{n+1}\right) + \frac{\lambda_1 h}{2a_1\tau_{n+1}}\left(T_{i^*}^{n+1} - T_{i^*}^{n}\right)$$

$$\lambda_2 \left.\frac{\partial T_2}{\partial x}\right|_{x=\xi}^{n+1} = \frac{\lambda_2}{h}\left(T_{i^*+1}^{n+1} - T_{i^*}^{n+1}\right) + \frac{\lambda_2 h}{2a_2\tau_{n+1}}\left(T_{i^*}^{n+1} - T_{i^*}^{n}\right)$$

O(h^2) xatosi bilan chegaraviy shartning (2) yaqinlashishi quyidagi shaklni oladi:

$$\frac{\lambda_1}{h}\left(T_{i^*}^{n+1} - T_{i^*-1}^{n+1}\right) + \frac{\lambda_1 h}{2a_1\tau_{n+1}}\left(T_{i^*}^{n+1} - T_{i^*}^{n}\right) - \frac{\lambda_2}{h}\left(T_{i^*+1}^{n+1} - T_{i^*}^{n+1}\right) + \frac{\lambda_2 h}{2a_2\tau_{n+1}}\left(T_{i^*}^{n+1} - T_{i^*}^{n}\right) = Q(1-m)\rho w \frac{h}{\tau_{n+1}};$$

$$\tau_{n+1} = \frac{2a_1a_2h^2 Q(1-m)\rho w(\lambda_1 a_2 + \lambda_2 a_1)\left(T_{i^*}^{n+1} - T_{i^*}^{n}\right)}{2a_1a_2\lambda_1\left[\left(T_{i^*}^{n+1} - T_{i^*-1}^{n+1}\right) - \lambda_2\left(T_{i^*+1}^{n+1} - T_{i^*}^{n+1}\right)\right]};$$

$$\tau_{n+1} = \frac{2a_1a_2h^2 Q(1-m)\rho w(\lambda_1 a_2 + \lambda_2 a_1)\left(T_f - T_{i^*}^{n}\right)}{2a_1a_2\lambda_1\left[\left(T_f - T_{i^*-1}^{n+1}\right) - \lambda_2\left(T_{i^*+1}^{n+1} - T_f\right)\right]}. \qquad (14)$$

FOYDALANILGAN ADABIYOTLAR RO‘YXATI

1. Самарский А.А. Вычислительная теплопередача / А.А. Самарский, П.К. Вабищевич – М.: Едиториал УРСС, 2003 – 784 с.

2. 3. Кузнецов Г.В. Разностные методы решения задач теплопроводности: учебное пособие/ Г.В. Кузнецов, М.А. Шеремет – Томск: Изд-во ТПУ, 2007. – 12 с.

3. Борисов В.С. Численное решение задачи о процессах промерзания и протаивания в многолетних грунтах – Вестник СВФУ, 2015 – Т. 12 – № 2. С. 36- 42.