MODAL SYNTHESIS OF CONTROL ALGORITHMS FOR MULTIDIMENSIONAL SYSTEMS WITH A GIVEN SPECTRUM WITH INCOMPLETE INFORMATION

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ABSTRACT

The modal synthesis of control algorithms for multidimensional systems with a given spectrum is considered under the condition of incomplete information about the state vector. For a linear multidimensional stationary system, a method is proposed for synthesizing systems with a given spectrum with incomplete information by designing a full-rank feedback matrix, in which the closed system has all eigenvalues equal to the given ones. The key elements in the considered expressions are pseudo-inverse matrices. For a multidimensional control system, a method is proposed for synthesizing systems with a given spectrum with incomplete information by applying a special canonical form. This algorithm provides the desired loop pole distribution for each channel.

Keywords: *linear system, control law, modal synthesis, matrix differential equation, control object, pseudoinverse matrices, pole placements.*

1. Introduction

Among the various directions in the theory of control of dynamic systems based on the state space method, two approaches can be distinguished that have received the greatest distribution in engineering practice [1–8]. One of them is based on methods for optimizing a dynamic system by minimizing a certain functional (usually an integral of some quadratic form) that characterizes the quality of control. The other one is connected with the methods of modal analysis and synthesis, i.e. methods of analyzing the dynamic properties of the system and the formation of feedback control laws that give the closed system a pre-selected distribution of eigenvalues.

At present, modal control is usually understood as the problem of designing controllers with a given spectrum, i.e. the problem of choosing such a feedback that provides a shift or the desired placement of all or only some poles of a closed system on the complex plane [1,5,7-9].

In practical problems [7,8,10,14], many well-known methods are not applicable due to their inherent disadvantages: poor conditionality of the matrices used (for example, controllability matrices); possible unsolvability of the problem under complete controllability (for example, restrictions on the multiplicity of assigned eigenvalues); rapid growth in the dimension of the equations being solved; the difficulty of constructing a set of equivalent controllers; the absence of a clear influence on other properties of a closed system (the quality of transient processes, robustness, etc.).

Thus, comparing the capabilities of the currently most developed methods for synthesizing linear systems with the requirements imposed on them by technological control objects, we can conclude that the modal control method meets them to the greatest extent. However, for its use in the tasks of automation of technological objects, it is necessary to solve a number of practical and theoretical problems related to the features of the mathematical description of models and the analysis of their properties, methods for ensuring the necessary properties of modal control systems in transient and static operating modes, methodological and software.

An analysis of the features of the functioning of typical technological objects and the requirements for them allows us to outline the range of tasks that need to be solved for the effective use of the modal control method [5,6,9-15], chosen as the basis for the synthesis of ACS.

In this paper, we propose a method for the synthesis of systems with a given spectrum by designing a full-rank feedback matrix, in which the closed system has all eigenvalues equal to the given ones.

2. Formulation of the problem

Consider a linear stationary system, described by a matrix differential equation:

$$\dot{\hat{x}} = \hat{A}\hat{x} + \hat{B}\hat{u}, \quad \hat{y} = \hat{H}\hat{x}, \qquad (1)$$

where $\hat{x}$ is a n-dimensional state vector, $\hat{u}$ is a m-dimensional control vector, $\hat{y}$ is a l-dimensional output vector, $\hat{A}, \hat{B}, \hat{H}$ are constant matrices of the corresponding dimensions. System (1) is assumed to be fully controllable, i.e. $\operatorname{rank}\left[\hat{B},\ \hat{A}\hat{B},\ \ldots, \hat{A}^{n-1}\hat{B}\right] = n$. It is also assumed that matrices $\hat{B}$ and $\hat{H}$ are of full rank.

It is necessary to introduce into system (1) a linear stationary feedback on the output

$$\hat{u} = \hat{K}\hat{y} = \hat{K}\hat{H}\hat{x} \qquad (2)$$

so that the closed system has a predetermined spectrum $\lambda_1, \lambda_2, \ldots, \lambda_n$ (λ_i is an eigenvalue of the matrix $\hat{A} + \hat{B}\hat{K}\hat{A}$, $\hat{K}$ is a feedback matrix of dimension $m \times l$).

3. Solution of the problem

The controllability of system (1) implies the existence of a nondegenerate matrix N of dimension $n \times n$ and a permutation matrix M of dimension $m \times m$ [12,13], such that the transformation

$$x = N\hat{x}, \quad u = M^{-1}\hat{u}, \quad y = \hat{y} \tag{3}$$

brings system (1) to the form

$$\dot{x} = Ax + Bu, \quad y = Hx. \tag{4}$$

Here matrices A, B and H have the following structure:

$$A = N\hat{A}N^{-1} = \begin{bmatrix} A_1 \\ \hdashline A_2 \end{bmatrix} = \begin{array}{c} \left[\begin{array}{ccccc} & A_{1,2} & & \cdots & \\ & & A_{2,3} & \cdots & \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ & & & \cdots & A_{v-1} \\ \hdashline A_{v,1} & A_{v,2} & A_{v,3} & \cdots & A_{v,v} \end{array}\right] \begin{array}{l} l_1 \\ l_2 \\ \vdots \\ l_{v-1} \\ l_v \end{array} \\ \begin{array}{ccccc} l_1 & l_2 & l_3 & \cdots & l_v \end{array} \end{array} \tag{5}$$

$$B = N\hat{B}M = \begin{bmatrix} 0 \\ \hdashline B_v \end{bmatrix}\begin{array}{l} {}^{n-m} \\ {}_{m} \end{array}, \tag{6}$$

$$H = \hat{H}N^{-1}, \tag{7}$$

where

$$A_{i,i+1} = [\underset{l_{i+1}-l_i}{0} \vdots \underset{l_i}{I}]l_i, \quad i = 1,2,\dots,v-1,$$

$$B_v = \begin{array}{c} \left[\begin{array}{cccc} I & 0 & \dots & 0 \\ \Delta & I & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \Delta & \Delta & \dots & I \end{array}\right] \begin{array}{l} l_v - l_{v-1} \\ \\ \vdots \\ l_1 \end{array} \\ \begin{array}{ccc} l_v - l_{v-1} & \dots & l_1 \end{array} \end{array}, \tag{8}$$

$A_{v,1}, A_{v,2}, \dots, A_{v,v}$, Δ are undefined submatrices, I is the identity matrix. Here, the controllability index v is equal to the smallest integer for which $\operatorname{rank}\left[\hat{B}, \ \hat{A}\hat{B}, \ \dots, \ \hat{A}^{v-1}\hat{B}\right] = n$. Sequence l_i is determined from the following expressions:

$$l_i = \operatorname{rank}\left[\hat{B}, \ \hat{A}\hat{B}, \ \dots, \ \hat{A}^{v-1}\hat{B}\right] - \operatorname{rank}\left[-\hat{B}, \hat{A}\hat{B} \dots, \hat{A}^{v-i-1}\hat{B}\right], \ (i = 1,2,\dots,v)$$

$$l_v = \operatorname{rank}\hat{B} = m, \quad \sum_{i=1}^{v} l_i = n.$$

The control for system (4) is sought in the form

$$u = Ky = KHx, \tag{9}$$

where $K = M^{-1}\hat{K}$. Let matrix S of dimension $(n-l) \times n$ be the orthogonal complement of matrix $H\left(HS' = 0, \operatorname{rank}S = n - l\right)$. We will assume that the matrix of the closed system

$C = A + BKH$ (10)

has the form

$$C = \begin{bmatrix} A_1 \\ \cdots \\ Z \end{bmatrix}, \quad (11)$$

where $((n-m)\times n)$-submatrix A_1 is determined from (5), $(m\times n)$-submatrix Z is set in such a way that matrix C has the desired spectrum.

If system (1) is controllable and its canonical representation (4) satisfies the condition

$$[Z - A_2]S' = 0, \quad (12)$$

then the spectrum of system (1) can be arbitrarily set using feedback of the form (2), and $\hat{K} = MB_v^{-1}[Z - A_2H^+]$ (matrix $H^+ = H'(HH')^{-1}$ is pseudoinverse [12-14] for matrix H, A_2 and B_v are determined from expressions (5), (6)).

The matrix of the closed system C is selected by the designer from the condition of fulfillment of the properties of the closed system that are of interest to him. If it is required that the matrix C has a given spectrum $\Gamma = \{\aleph_1,...,\aleph_n\}$, Γ is a set of n real or complex conjugate numbers characterizing the quality of the closed system, then the given polynomial should be added to conditions (12)

$$\det(C - \lambda I) = c_n^* + c_1^*\lambda + ... + c_{n-1}^*\lambda^{n-1} + \lambda^n, \quad (13)$$

where $c_n^* + c_1^*\lambda + ... + c_{n-1}^*\lambda^{n-1} + \lambda^n$ is a given characteristic polynomial.

If $\aleph_i \neq \aleph_j$ at $i \neq j$, (12) will be written in the form

$$B^{\perp'}\left(A - S^{-1}\mathrm{diag}(\aleph_1,...,\aleph_n)S\right) = 0, \quad (14)$$

$$\left(A - S^{-1}\mathrm{diag}(\aleph_1,...,\aleph_n)S\right)H^{\perp'} = 0. \quad (15)$$

If system (14), (15) is satisfied by some nonsingular matrix S, then the problem is solvable.

Consider case $\mathrm{rank}B = n$.

Then (14) is fulfilled identically and the problem is reduced to the consideration of the matrix equation [13]

$$SA\left(H^{\perp'}\right)' - \mathrm{diag}(\aleph_1,...,\aleph_n)S\left(H^{\perp}\right)' = 0, \quad (16)$$

which breaks down into n independent subsystems

$$H^{\perp}(A' - \aleph_i I)\begin{pmatrix} s_{i1} \\ \vdots \\ s_{in} \end{pmatrix} = 0, \quad (i = 1,...,n), \quad (17)$$

whose solutions with respect to must be linearly independent.

In conclusion, we give a geometric interpretation of the problem under consideration.

Let $R = \mathrm{diag}(\aleph_1,...,\aleph_n)$, i.e. $C = S^{-1}RS$.

Condition (10) can be written as

$$SA + SBKH = RS. \tag{18}$$

Let us rewrite (18) using the Kronecker products and the notation $vecK = (k_{11},...,k_{1n};...,k_{m1},...,k_{mn})'$ [12,13]:

$$(B \otimes I)(I \otimes H')vecK = vec(S^{-1}RS) - vecA. \tag{19}$$

In n^2 - dimensional space, the right side of (19) determines the locus of all similar matrices, transferred in parallel to $vecA$ (set I).

The left side for all possible K determines the plane spanned by the columns of the matrix $(B \otimes I)(I \otimes H') = B \otimes H'$ (set II).

If the intersection of sets I and II is not empty, then the problem is solvable.

4. Conclusion

The results of the analysis confirmed their effectiveness, which makes it possible to use them in solving applied problems of modal control of multidimensional systems with a given spectrum with incomplete information and in the synthesis of process control systems.

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