SILJISHLI FUNKSIONAL OPERATORLARNING SILJISH YO‘NALISHNI O‘ZGARTUVCHI BO‘LGAN HOLDA BIR TOMONLAMA TESKARILANUVCHANLIK SHARTLARI

Mardiyev Rasul
Jumanov Xudoyor Shuxrat o‘g‘li
Samarqand davlat universiteti

Annotatsiya: Siljish yo‘nalishni o‘zgartuvchi bo‘lgan holda siljishli funksional operatorlarning $H_\mu(\Gamma), 0 < \mu < 1$ Gyolder fazosining qism fazosi bo‘lgan

$$H_\mu^0(\Gamma, \Lambda) = \{ \varphi \in H_\mu(\Gamma) \colon \varphi(\tau) \in N_0 \}$$

fazoda bir tomonlama teskrilanuvchanlik kriteriysi olingan. Bu yerda Γ- sodda silliq yopiq kontur α - Γ konturni o‘ziga akslantiruvchi diffeomorfizm (siljish) Λ- $\alpha_2(t) = \alpha(\alpha(t))$ siljishning qo‘zg‘almas nuqtalari to‘plami, $\Phi = supp[\tau - \alpha_m(\tau)]$ - Γ konturning $\alpha_2(\tau) \neq \tau$ shartni qanoatlantiruvchi nuqtalar to‘plamining yopig‘i, . $\mathrm{N}_0 = \{ \Lambda \cap \Phi \}$.

Kalit so‘zlar: Diffeomorfizm, siljishli funksional operator, bir tomonlama teskarilanuvchanlik shartlari, yo‘nalishni saqlovchi bo‘lgan siljishli funksional operatorlar, o‘ngdan (chapdan) teskarilanuvchanlik, $H_\mu^0(\Gamma, \mathrm{N}_0)$ va $(H_\mu^0(\Gamma, \mathrm{N}_0))^2$ fazolar, $\mathbb{d}$ ($\mathbb{m}$) - normallanganligi, $B(\mathbb{C})$ operator

Γ- sodda silliq kontur, α - Γ konturni o‘ziga akslantiruvchi diffeomorfizm (siljish) bo’lsin.

Bizga ma’lumki (masalan, [1], 24-29-betga qarang) yo‘nalishni o‘zgartuvchi siljish albatta ikkita z_1 va z_2 qo‘zg‘almas nuqtaga ega bo‘ladi.

Γ konturning z_1 va z_2 nuqtalarni z_1 nuqtadan z_2 nuqtaga musbat yo‘nalishda olingan yoyni Υ_1 orqali belgilaymiz, $\Upsilon_2 = \Gamma \backslash \Upsilon_1$

Bu ishda Gyolder fazolarida quyidagi ko‘rinishdagi siljishli funksional operatorning bir tomonlama teskarilanuvchanlik shartlari o‘rganiladi

$$A = a(t)I - b(t)W \qquad (1)$$

bu yerda $a(t), b(t) \in H_\mu(\Gamma)$, $I -$ birlik operator, W- siljish operatori: $(W\varphi)(t) = \varphi[\alpha(t)], t \in \Gamma$.

A operatorni $\alpha(t)$ siljish yo‘nalishni o‘zgartuvchi bo‘lgan holda o‘rganish bu operatorni yo‘nalishni saqlovchi bo‘lgan $\alpha_2(t) = \alpha(\alpha(t))$ siljishli funksional operatorlarni o‘rganishga keltiriladi.

Faraz qilaylik $\alpha_2(t)$ siljish ixtiyoriy sondagi qo‘zg‘almas nuqtalarga ega bo‘lsin. Qo‘zg‘almas nuqtalar to‘plamini Λ orqali belgilaymiz.

$\Phi = supp[\tau - \alpha_m(\tau)]$ orqali Γ konturning $\alpha_2(\tau) \neq \tau$ shartni qanoatlantiruvchi barcha nuqtalar to‘plamining yopig‘ini belgilaymiz.

$N_0 = \{ \Lambda \cap supp[\tau - \alpha_m(\tau)] \}$ bo‘lsin.

$H_\mu^0(\Gamma, N_0) \stackrel{\text{def}}{=} \{\varphi \in H_\mu(\Gamma): \varphi(\tau) = 0, \tau \in N_0 \}$ fazoda A operatorni qaraymiz.

[2] adabiyotdagi 2.3-lemmaga asosan quyidagi tasdiq isbotlanadi.

1-lemma: A operator $H_\mu^0(\Gamma, N_0)$ fazoda o‘ngdan (chapdan) teskarilanuvchi bo‘lishi uchun

$$\tilde{A} = \begin{pmatrix} a(t)I & -b(t)I \\ -b(\alpha(t))W^2 & a(\alpha(t))I \end{pmatrix}$$

operatorning $(H_\mu^0(\Gamma, N_0))^2$ fazoda o‘ngdan (chapdan) teskarilanuvchan bo‘lishi zarur va yetarli

A operator uchun quyidagi tasdiqlarni isbotlaymiz.

2-lemma: A operator $H_\mu^0(\Gamma, N_0)$ fazoda o‘ngdan teskarilanuvchi bo‘lsa, u holda

$$\inf_{t\in\Gamma}\{ |a(t)| + |b(t)| \} > 0 \qquad (2)$$

tengsizlik o‘rinli.

Isboti: Agar (2) tengsizlik o‘rinli bo‘lmasa, u holda shunday $t_0 \in \Gamma$ nuqta topilib $a(t_0) = b(t_0) = 0$ tenglik o‘rinli bo‘ladi. U holda $\tilde{A}$ operator uchun quyidagi munosabat o‘rinli

$$\begin{pmatrix} aI & -bI \\ -b(\alpha)W^2 & a(\alpha)I \end{pmatrix} = \begin{pmatrix} |t-t_0|^{\mu}I & 0 \\ 0 & I \end{pmatrix}\begin{pmatrix} |t-t_0|^{-\mu}a\Pi_{t_0} & -|t-t_0|^{-\mu}b\Pi_{t_0} \\ -b(\alpha)W^2 & a(\alpha)I \end{pmatrix} \quad (3)$$

bu yerda $(\Pi_{t_0}\varphi)(t) = \varphi(t) - \varphi(t_0)$. Ko‘rinib turibdiki

$$Y = \begin{pmatrix} |t-t_0|^{-\mu}a\Pi_{t_0} & -|t-t_0|^{-\mu}b\Pi_{t_0} \\ -b(\alpha)W^2 & a(\alpha)I \end{pmatrix}$$

operator $H_{\mu}^{0}(\Gamma, \mathrm{N}_0)$ fazoda chegaralangan va 1-lemmaga asosan $\tilde{A}$ operator o‘ngdan teskarilanuvchi. Demak, u $\mathbb{d}^*$- normal operator bo’ladi. U holda (3) dan ([3]. 195-bet)

$$E = \begin{pmatrix} |t-t_0|^{\mu}I & 0 \\ 0 & I \end{pmatrix}$$

operatorning $(H_{\mu}^{0}(\Gamma, \mathrm{N}_0))^2$ fazoda $\mathbb{d}$ - normalligi kelib chiqadi. E operator dioganal ko‘rinishda bo‘lganligi uchun $\mathbb{d}$ - normalligidan $K = |t-t_0|^{\mu}I$ operatorning $H_{\mu}^{0}(\Gamma, \mathrm{N}_0)$ fazoda $\mathbb{d}$ - normalligi kelib chiqadi. Ammo ossonlik bilan ko‘rish mumkinki bu operatorning $\mathbb{d}$- normalligi uning o‘ngdan teskarilanuvchanligiga ekvivalent. Ma’lumki K operator o‘ngdan teskarilnuvchan emas. Bu qarama-qarshilik lemmani isbotlaydi

Agar (2) shart bajarilsa, u holda shunday $g(t) \in H_{\mu}(\Gamma, \mathrm{N}_0)$ funksiya topilib, Γ konturning barcha nuqtalarida $f(t) = a(t)g(t) + b(t) \neq 0$ shart bajariladi va $\tilde{A}$ operatorni

$$\tilde{A} = \begin{pmatrix} I & 0 \\ -(WbWgI + WaW^{-1})f^{-1}I & -f^{-1}\big(aa(\alpha)I - b(\alpha)b(\alpha_2)\big)ff^{-1}(\alpha_2)W \end{pmatrix} \cdot$$

$$\cdot \begin{pmatrix} aI & -bI \\ I & gI \end{pmatrix} \quad (4)$$

ko‘rinishda tasvirlash mumkin. Ixtiyoriy $t \in \Gamma$ uchun $f(t) \neq 0$ bo‘lganligi uchun (4) ning ikkinchi ko‘paytuvchisi teskarilanuvchi. Demak, $\tilde{A}$ operatorning o‘ngdan teskarilanuvchanligi

$$\tilde{B} = a(t)a\big(\alpha(t)\big)I - b(\alpha(t)) \cdot B\big(\alpha_2(t)\big)f(t)f^{-1}(\alpha_2(t))W$$

operatorning o‘ngdan teskarilanuvchanligiga ekvivalent bo‘ladi.

$W^2 = W_{\alpha_2}$ va α_2 Γ konturning yo‘nalishini saqlaganligi uchun [4] ishdagi teoremadan foydalanib, $\tilde{B}$ operator $H_\mu^0(\Gamma, \mathrm{N}_0)$ fazoda o‘ngdan teskarilanuvchanligi $B = a(t)a(\alpha(t))I - b(\alpha(t))b(\alpha_2(t))W^2$ operatorning $H_\mu^0(\Gamma, \mathrm{N}_0)$ fazoda o‘ngdan teskarilanuvchanligiga va (2) shartning bajarilishiga ekvivalent ekanligini ko‘rsatish mumkin.

Endi A operatorning $H_\mu^0(\Gamma, \mathrm{N}_0)$ fazoda chapdan teskarilanuvchanlik shartlarini [62] o‘rganamiz.

3-lemma: A operator $H_\mu^0(\Gamma, \mathrm{N}_0)$ fazoda chapdan teskarilanuvchi bo‘lsa, u holda

$$\inf_{t\in\Gamma}\{\, |a(\alpha(t))| + |b(t)| \,\} > 0 \tag{5}$$

tengsizlik o‘rinli.

Isboti: Faraz qilaylik (5) tengsizlik o‘rinli bo‘lmasin, u holda shunday $t_0 \in \Gamma$ nuqta topilib $a(t_0) = b(t_0) = 0$ tenglik o‘rinli bo‘ladi va $\tilde{A}$ operator uchun quyidagi munosabat o‘rinli

$$\tilde{A} = \begin{pmatrix} aI & b(t)|t-t_0|^{-\mu}\Pi_{t_0} \\ -b(\alpha)W^2 & a(\alpha)|t-t_0|^{-\mu}\Pi_{t_0} \end{pmatrix} \cdot \begin{pmatrix} I & 0 \\ 0 & |t-t_0|^{\mu} I \end{pmatrix} \tag{6}$$

bu yerda $(\Pi_{t_0}\varphi)(t) = \varphi(t) - \varphi(t_0)$.

A operatorning $H_\mu^0(\Gamma, \mathrm{N}_0)$ fazoda chapdan teskarilanuvchanligi $\tilde{A}$ operatorning $(H_\mu^0(\Gamma, \mathrm{N}_0))^2$ fazoda chapdan teskarilanuvchanligiga (1-lemmaga asosan) aekvivalent bo‘lganligi uchun (6) dan

$$E_1 = \begin{pmatrix} I & 0 \\ 0 & |t-t_0|^{\mu} I \end{pmatrix}$$

operatorning $(H_\mu^0(\Gamma, \mathrm{N}_0))^2$ fazoda $\mathfrak{m}$- normallanganligi ([3]. 196-bet) kelib chiqadi. Ammo E_1 operatorning $(H_\mu^0(\Gamma, \mathrm{N}_0))^2$ fazoda $\mathfrak{m}$- normallanganligidan $K = |t-t_0|^{\mu} I$ operatorning $H_\mu^0(\Gamma, \mathrm{N}_0)$ fazoda $\mathfrak{m}$- normallanganligi kelib chiqadi. Ammo ossonlik bilan ko‘rish mumkinki K operatorning $H_\mu^0(\Gamma, \mathrm{N}_0)$ fazoda $\mathfrak{m}$- normalligi uning chapdan teskarilanuvchanligiga ekvivalent. Ma’lumki K operator

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*Agar A operatorning obrazi yopiq va koyadrosi (yadrosi) chekli o‘lchovli bo‘lsa, u holda A operator $\mathfrak{d}$ ($\mathfrak{m}$) normal deyiladi

$H_\mu^0(\Gamma, \mathrm{N}_0)$ fazoda chapdan teskarilanuvchi emas. Bu olingan qarama-qarshilik lemmani isbotlaydi.

Ossonlik bilan koʻrish mumkinki (5) shart bajarilganda shunday $p(t) \in H_\mu(\Gamma)$ funksiya topilib Γ konturning barcha nuqtalarida $\varphi(t) = p(t)a\big(\alpha(t)\big) + b(t) \neq 0$ shart bajariladi va $\tilde{A}$ operatorni quyidagicha tasvirlash mumkin

$$\tilde{A} = \begin{pmatrix} p(t)I & -b(t)I \\ I & a\big(\alpha(t)\big)I \end{pmatrix} \begin{pmatrix} \varphi^{-1}\big(aa(\alpha)\big)I - bb(\alpha)W^2 & 0 \\ -\varphi^{-1}(a)I + pb(\alpha)W^2 & I \end{pmatrix} \qquad (7)$$

Γ konturning barcha nuqtalarida $\varphi(t) \neq 0$ boʻlganligi uchun (7) dagi birinchi koʻpaytuvchi teskarilanuvchi. Demak, $\tilde{A}$ operatorning chapdan teskarilanuvchanligi $\mathbb{C} = aa(\alpha)I - bb(\alpha)W^2$ operatorning $H_\mu^0(\Gamma, \mathrm{N}_0)$ fazoda chapdan teskarilanuvchanligiga ekvivalent boʻladi. Natijada A operatorning $H_\mu^0(\Gamma, \mathrm{N}_0)$ fazoda chapdan teskarilanuvchanligi $\mathbb{C}$ operatorning $H_\mu^0(\Gamma, \mathrm{N}_0)$ fazoda chapdan teskarilanuvchanligiga va (5) shartning bajarilishiga ekvivalent ekanligini olamiz.

Shunday qilib biz quyidagi teoremaning oʻrinli ekanligini isbotladik.

1-teorema: A operator $H_\mu^0(\Gamma, \mathrm{N}_0)$ fazoda oʻngdan (chapdan) teskarilanuvchi boʻlishi uchun $B(\mathbb{C})$ operatorning $H_\mu^0(\Gamma, \mathrm{N}_0)$ fazoda oʻngdan (chapdan) teskarilanuvchi boʻlishi va (2)((5)) shartning bajarilishi zarur va yetarli.

Endi $B(\mathbb{C})$ operatorni $H_\mu^0(\Gamma, \mathrm{N}_0)$ fazoda (2)((3)) shartlar bajarilganda oʻrganamiz. [4] ishdagi belgilashlardan foydalangan holda quyidagi belgilashlarni kiritamiz. $u(t), a(t), b(t)$ funksiyalari uchun quyidagi belgilashlarni kiritamiz

$$u_\pm(t) = \lim_{n\to\pm\infty} u\big(\alpha_n(t)\big) \cdot u\big(\alpha_{n+1}(t)\big), h_\pm(t) = \big|a_\pm(t)\big| - \big|\alpha'_\pm(t)\big|^{-\frac{1}{p}}\big|b_\pm(t)\big|,$$

$$\Gamma_1 = \Gamma\backslash\Phi, \Gamma_2 = \{\, t \in \Phi: h_\pm(t) > 0 \,\}, \Gamma_3 = \{\, t \in \Phi: h_\pm(t) < 0 \,\},$$

$$\Gamma_4 = \{\, t \in \Phi: h_+(t) < 0 < \ h_-(t)\}, \Gamma_5 = \{\, t \in \Phi: h_+(t) > 0 > \ h_-(t)\},$$

$$\nu_A(t) = \begin{cases} a(t)a\big(\alpha(t)\big) - b(t)b\big(\alpha(t)\big) & , t \in \Gamma_1 \\ a(t)a\big(\alpha(t)\big) & , t \in \Gamma_2 \\ b(t)b\big(\alpha(t)\big) & , t \in \Gamma_3 \\ 0 & , t \in \Gamma\backslash\bigcup_{i=1}^{3}\Gamma_i \end{cases}$$

$B(\mathbb{C})$ operatorlar to‘g‘ri siljishli (yo‘nalishni saqlovchi) operatorlar bo‘lganligi uchun [4] ishdagi teorema isbotiga o‘xshash quyidagi teoremani isbotlash mumkin.

2-teorema: A operator $H_{\mu}^{0}(\Gamma, \mathrm{N}_0)$ fazoda o‘ngdan (chapdan) teskarilanuvchi bo‘lishi uchun

$$\nu_A(t) \neq 0, \forall t \in \Gamma \backslash \Gamma_4 \ \ (\forall t \in \Gamma \backslash \Gamma_5) \qquad (8)$$

va Γ_4 (Γ_5) to‘plamda

$(\forall t \in \Gamma_4)$ $(\exists k_0 \in \mathbb{Z})$ $k \geq k_0$ bo‘lganda $b(\alpha_k(t)) \neq 0$

$k < k_0$ bo‘lganda $a(\alpha_k(t)) \neq 0$

(mos ravishda

$(\forall t \in \Gamma_5)$ $(\exists k_0 \in \mathbb{Z})$ $k < k_0$ bo‘lganda $b(\alpha_k(t)) \neq 0$

$k > k_0$ bo‘lganda $a(\alpha_k(t)) \neq 0$).

shartlarning bajarilishi zarur va yetarli.

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