MATEMATIKANING QIZIQARLI MISOLLARI

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Annotatsiya: Ushbu maqolada matematika qiziquvchilari bo`lmish o`quvchilar uchun ajoyib misollar va masala yechimlarini sodda tartibda ishlash va tushuntirish maqsad qilingan. Bunday misollarni yechishda matematikadagi teorema va formulalardan to`liq foydalaniladi.

Kalit so`zlar: ildiz, sonning butun qismi, tengsizlik, kvadrat ildiz.

1. Soddalashtiring: $\frac{\sqrt{2}+\sqrt{3}+\sqrt{4}}{\sqrt{2}+\sqrt{3}+\sqrt{6}+\sqrt{8}+4}$

Yechish:

$$\frac{\sqrt{2}+\sqrt{3}+\sqrt{4}}{\sqrt{2}+\sqrt{3}+\sqrt{6}+\sqrt{8}+4}=\frac{\sqrt{2}+\sqrt{3}+2}{\sqrt{2}+\sqrt{3}+2+\sqrt{2}\cdot\sqrt{3}+2\sqrt{2}+2}=$$

$$=\frac{\sqrt{2}+\sqrt{3}+2}{\sqrt{2}+\sqrt{3}+2+\sqrt{2}\left(\sqrt{2}+\sqrt{3}+2\right)}=\frac{\sqrt{2}+\sqrt{3}+2}{\left(\sqrt{2}+\sqrt{3}+2\right)\cdot\left(1+\sqrt{2}\right)}=$$

$$=\frac{1}{1+\sqrt{2}}=\frac{1-\sqrt{2}}{\left(1+\sqrt{2}\right)\cdot\left(1-\sqrt{2}\right)}=\frac{1-\sqrt{2}}{-1}=\sqrt{2}-1$$

Javob: $\sqrt{2}-1$

2. Yig`indini toping: $\frac{1}{1+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{5}}+\frac{1}{\sqrt{5}+\sqrt{7}}+...+\frac{1}{\sqrt{79}+\sqrt{81}}$

Yechish:

$$\frac{1}{1+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{5}}+\frac{1}{\sqrt{5}+\sqrt{7}}+...+\frac{1}{\sqrt{79}+\sqrt{81}}=$$
$$=\frac{\sqrt{3}-1}{\left(1+\sqrt{3}\right)\left(\sqrt{3}-1\right)}+\frac{\sqrt{5}-\sqrt{3}}{\left(\sqrt{3}+\sqrt{5}\right)\left(\sqrt{5}-\sqrt{3}\right)}+\frac{\sqrt{7}-\sqrt{5}}{\left(\sqrt{5}+\sqrt{7}\right)\left(\sqrt{7}-\sqrt{5}\right)}+...$$
$$...+\frac{\sqrt{81}-\sqrt{79}}{\left(\sqrt{79}+\sqrt{81}\right)\left(\sqrt{81}-\sqrt{79}\right)}=\frac{\sqrt{3}-1}{2}+\frac{\sqrt{5}-\sqrt{3}}{2}+\frac{\sqrt{7}-\sqrt{5}}{2}+...+\frac{\sqrt{81}-\sqrt{79}}{2}=$$
$$=\frac{\sqrt{81}-1}{2}=\frac{9-1}{2}=\frac{8}{2}=4$$

Javob: 4

3. Tenglamani yeching: $\left[\sqrt[3]{1}\right]+\left[\sqrt[3]{2}\right]+...+\left[\sqrt[3]{x^3-1}\right]=400$

Yechish: $\left[\sqrt[3]{1}\right]+\left[\sqrt[3]{2}\right]+...+\left[\sqrt[3]{x^3-1}\right]=400$

Tenglamada quyidagi $\left[\sqrt[3]{1}\right]$;…; $\left[\sqrt[3]{7}\right]$ ($\left[\sqrt[3]{7}\right]$ ham qabul qiladi) sonlarning qiymati 1 ga teng. $\left[\sqrt[3]{8}\right]$;…;$\left[\sqrt[3]{26}\right]$ sonlarning qiymati esa 2 ga teng. $\left[\sqrt[3]{27}\right]$;…; $\left[\sqrt[3]{63}\right]$ sonlarning qiymati esa 3 ga teng, $\left[\sqrt[3]{64}\right]$;…; $\left[\sqrt[3]{124}\right]$ sonlarning qiymati esa 4 ga teng. Ushbu butun sonlar yig`indisini hisoblaymiz. Bunda 7 ta 1, 26-7=19 ta 2, 63-26=37 ta 3 va 124-63=61 ta 4 bo’ladi. $7\cdot 1+19\cdot 2+37\cdot 3+60\cdot 4=400$. Demak, $\left[\sqrt[3]{x^3-1}\right]=4$ son eng oxirgi qo`shiluvchi bo`ladi, ya`ni $\left[\sqrt[3]{124}\right]$ sonidir.

$$\left[\sqrt[3]{x^3-1}\right]=\left[\sqrt[3]{124}\right]$$
$$\left(\left[\sqrt[3]{x^3-1}\right]\right)^3=\left(\left[\sqrt[3]{124}\right]\right)^3$$
$$x^3-1=124$$
$$x^3=125$$
$$x=5$$

Javob: x=5

4. Noldan farqli a,b,c sonlar uchun a+b+c=26, $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=28$ ***bo`lsa,***

$\frac{a}{b}+\frac{b}{c}+\frac{c}{a}+\frac{a}{c}+\frac{c}{b}+\frac{b}{a}$ ***ni toping.***

Yechish: a+b+c=26, $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=28$

$$a+b=26-c$$
$$a+c=26-b$$
$$b+c=26-a$$

$$\frac{a}{b}+\frac{b}{c}+\frac{c}{a}+\frac{a}{c}+\frac{c}{b}+\frac{b}{a}=\frac{b+c}{a}+\frac{a+c}{b}+\frac{a+b}{c}=\frac{26-a}{a}+\frac{26-b}{b}+\frac{26-c}{c}$$

$$\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=\frac{bc+ac+ab}{abc}=28$$

$$bc+ac+ab=28abc$$

$$\frac{26-a}{a}+\frac{26-b}{b}+\frac{26-c}{c}=\frac{(26-a)bc+(26-b)ac+(26-c)ab}{abc}=$$

$$=\frac{26bc-abc+26ac-abc+26ab-abc}{abc}=\frac{26bc+26ac+26ab-3abc}{abc}=$$

$$=\frac{26(bc+ac+ab)-3abc}{abc}=\frac{26\cdot 28abc-3abc}{abc}=\frac{abc(26\cdot 28-3)}{abc}=$$

$$=26\cdot 28-3=728-3=725$$

Javob: 725

5. Ixtiyoriy ABC uchburchak uchun $h_a \le \sqrt{p(p-a)}$ tengsizlik to’g’ri bo’lishini isbotlang.

Yechish:

Isboti: Uchburchak tomonlarini a, b, c deb belgilaymiz.

$$h_a \le \sqrt{p(p-a)};$$

$$h_a \le \sqrt{\frac{a+b+c}{2} \cdot \left(\frac{a+b+c}{2} - a\right)}$$

$$h_a \le \sqrt{\frac{a+b+c}{2} \cdot \frac{b+c-a}{2}};$$

$$h_a \le \frac{1}{2}\sqrt{(a+b+c)\cdot(b+c-a)};$$

$$2h_a \le \sqrt{(c+b)^2 - a^2};$$

$$h_a = \frac{2}{a}\sqrt{p(p-a)(p-b)(p-c)}$$

$$ah_a = 2\sqrt{\frac{a+b+c}{2} \cdot \left(\frac{a+b+c}{2} - a\right) \cdot \left(\frac{a+b+c}{2} - b\right) \cdot \left(\frac{a+b+c}{2} - c\right)}$$

$$ah_a = 2\sqrt{\frac{a+b+c}{2} \cdot \frac{a+b+c-2a}{2} \cdot \frac{a+b+c-2b}{2} \cdot \frac{a+b+c-2c}{2}}$$

$$ah_a = 2\sqrt{\frac{a+b+c}{2} \cdot \frac{b+c-a}{2} \cdot \frac{a+c-b}{2} \cdot \frac{a+b-c}{2}}$$

$$ah_a = 2 \cdot \frac{1}{4}\sqrt{\left((b+c)^2 - a^2\right)(a+c-b)(a+b-c)}$$

$$2ah_a = \sqrt{(b+c)^2 - a^2} \cdot \sqrt{(a+c-b)(a+b-c)};$$

$$\frac{2ah_a = \sqrt{(b+c)^2 - a^2} \cdot \sqrt{(a+c-b)(a+b-c)}}{2h_a \le \sqrt{(c+b)^2 - a^2}}$$

$$a \ge \sqrt{(a+c-b)(a+b-c)}$$

$$a^2 \ge \left(\sqrt{a^2 + ab - ac + ac + bc - c^2 - ab - b^2 + bc}\right)^2$$

$$a^2 \ge a^2 - \left(c^2 - 2bc + b^2\right)$$

$$a^2 \ge a^2 - (c-b)^2$$

Har qanday sonning kvadrati musbat. Shuning uchun bir sondan musbat sonni ayirsak undan kichik son hosil bo'ladi.

Javob: Isbotlandi.

Foydalanilgan adabiyotlar ro`yxati:

1. "Qiziqarli matematika va olimpiada masalalari"- A.S.Yunusov, S.I.Afonina, M.A.Berdiqulov, D.I.Yunusova. Toshkent-2007y;

2. *olympiad.uzedu.uz*-sayti.