GLOBAL SOLVABILITY SOLUTIONS OF A CROSS-DIFFUSION PARABOLIC SYSTEM

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Abstract: In this paper, we study the properties of self-similar solutions of a cross-diffusion parabolic system. In particular, we find the self-similar supersolution and subsolution to obtain the critical global existence curve.

Key words: cross-diffusive system, Cauchy problem, global solvability, comparison principle.

Consider the following doubly parabolic equations

$$\begin{cases} \dfrac{\partial u}{\partial t} = \dfrac{\partial}{\partial x}\left(v^{m_1 - 1} \left| \dfrac{\partial u^k}{\partial x} \right|^{p-2} \dfrac{\partial u}{\partial x} \right) \\ \dfrac{\partial v}{\partial t} = \dfrac{\partial}{\partial x}\left(u^{m_2 - 1} \left| \dfrac{\partial v^k}{\partial x} \right|^{p-2} \dfrac{\partial v}{\partial x} \right) \end{cases} \qquad (1)$$

coupled via

$$\begin{aligned} & u(0,x) = u_0(x), \\ & v(0,x) = v_0(x), \qquad x > 0 \end{aligned} \qquad (2)$$

where $m_i, k, p, q_i \ (i = 1, 2)$ are numerical parameters, and u_0, v_0 are nonnegative continuous functions with compact support in $\mathbb{R}_+$.

Many processes in nature are represented by equations and systems of equations of the parabolic type using the Cauchy problem. Mathematical problem (1)-(2) used for describe different nonlinear process of biological population, chemical reactions, diffusion and etc. Many scientists have conducted their own research on this type of parabolic equations and systems of equations (1)-(2). In particular, there is a growing demand for solving Cauchy problems and studying its properties in non-divergent parabolic type systems (for example see [1],[3] and literature therein).

Here, we provide a method of nonlinear splitting for construction of self-similar equation for the system. We look for the solutions in the form:

$$u(t,x)=t^{-\alpha_1}f(\xi),$$
$$v(t,x)=t^{-\alpha_2}\psi(\xi)$$

where $\xi=\frac{x}{t^{\beta}}$.

Let`s do some calculations

$$\frac{\partial\xi}{\partial x}=t^{-\beta}$$

$$\frac{\partial\xi}{\partial t}=-\beta\frac{\xi}{t}$$

$$\frac{\partial u}{\partial t}=-\alpha_1 t^{-\alpha_1-1}f(\xi)+t^{-\alpha_1}\frac{\partial f}{\partial\xi}\cdot\left(-\beta\frac{\xi}{t}\right)=-t^{-\alpha_1-1}\left(\alpha_1 f+\beta\xi\frac{\partial f}{\partial\xi}\right)$$

$$\frac{\partial u}{\partial x}=t^{-\alpha_1-\beta}\frac{\partial f}{\partial\xi}$$

$$\frac{\partial u^k}{\partial x}=t^{-\alpha_1 k-\beta}\frac{\partial f^k}{\partial\xi}$$

$$\frac{\partial}{\partial x}=\frac{\partial}{\partial\xi}\frac{\partial\xi}{\partial x}=t^{-\beta}\frac{\partial}{\partial\xi}$$

The same calculations will be done for ψ function. After that, we put these calculations to the system (1), and we get the following a self-similar system of equations:

$$\begin{cases}\frac{d}{d\xi}\left(\psi^{m_1-1}\cdot\left|\frac{df^k}{d\xi}\right|^{p-2}\cdot\frac{df}{d\xi}\right)+\beta\xi\frac{df}{d\xi}+\alpha_1 f=0\\ \frac{d}{d\xi}\left(f^{m_2-1}\cdot\left|\frac{d\psi^k}{d\xi}\right|^{p-2}\cdot\frac{d\psi}{d\xi}\right)+\beta\xi\frac{d\psi}{d\xi}+\alpha_2\psi=0\end{cases}$$

where

$$\alpha_i = \frac{(1-\beta p)\left(k(p-2)+m_i-1\right)}{\Delta}, \quad (i=1,2)$$

$$\Delta = (k(p-2))^2 - \left(m_1-1\right)\left(m_2-1\right) \neq 0$$

Slow diffusion. A global solvability of solutions.

We obtain the properties of a global solvability for weak solutions of the system (1) by using a comparison principle.

A self-similar system of equations was constructed using the standard equation method as in:

$$u_+(t,x) = (t+T)^{-\alpha_1}\bar{f}(\xi)$$

$$v_+(t,x) = (t+T)^{-\alpha_2}\bar{\psi}(\xi)$$

We note that the functions:

$$\bar{f}(\xi) = A_1\left(a-\xi^{\gamma_0}\right)^{\gamma_1}$$

$$\bar{\psi}(\xi) = A_2\left(a-\xi^{\gamma_0}\right)^{\gamma_2}$$

where

$a>0,$

$$A_1^{k(p-2)} \cdot A_2^{m_i-1} = -\frac{\beta\gamma_0\gamma_1+\alpha_i}{\gamma_0\gamma_1|\gamma_0\gamma_1|^{p-2}k^{p-2}\left(1+\gamma_0\gamma_1\right)},$$

$$\gamma_0 = \frac{p}{p-1},$$

$$\gamma_i = \frac{(p-1)\left(k(p-2)+1-m_i\right)}{\left(k(p-2)^2-\left(m_1-1\right)\left(m_2-1\right)\right)}$$

Theorem. (A global solvability) Let the conditions of $\gamma_i>0$, $\beta \geq max\left(\alpha_1,\alpha_2\right)$

$$u_+(0,x) \geq u_0(x)$$

$$v_+(0,x) \geq v_0(x) \qquad x \in R^N$$

Then, for sufficiently small $u_0(x)$, $v_0(x)$ the following holds

$u(t,x) \leq u_{+}(t,x)$

$v(t,x) \leq v_{+}(t,x)$

in Q.

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