VOL’TERRA INTEGRAL TENGLAMALARIGA KELADIGAN BA’ZI MASALALAR

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Annotatsiya: Ushbu maqolada ba’zi fizik masalalar Abel va Koshi deb nomlanuvchi Vol’terra integral tenglamalariga keladigan isbotlar ko‘rib chiqiladi.

Kalit so‘zlar: Vol’terra integral, Abel, moddiy nuqta, egri chiziq, vertikal, yoy elementi, Koshi, uzluksiz funksiya

SOME ISSUES CONCERNING VOLTERRA INTEGRAL EQUATIONS

Abstract: This paper discusses some physical problems that come to the Volterra integral equations known as Abel and Cauchy.

Key words: Voltara integral, Abel, material point, curve, vertical, arc element, Cauchy, continuous function

I.Abel masalasi: Moddiy nuqta o‘zining og‘irlik kuchi ta’siri ostida vertikal tekislikda joylashgan biror silliq egri chiziq bo‘lib uning ordinatasi h ga teng bo‘lgan ixtiyoriy M nuqtasidan ordinatasi 0 ga teng bo‘lgan eng quyi O nuqtasigacha boshlang‘ich tezliksiz $T = \psi(h)$ ($\psi(h)$ – berilgan funksiya) vaqt ichida yetib kelgan bo‘lsin. Shu egri chiziqni toping.

Yechish: izlangan egri chiziqning eng quyi *O* nuqtasini koordinatalar boshi uchun qabul qilamiz va *OX* o‘qni gorizantal, *OY* o‘qni esa vertikal yo‘naltiramiz. Bu egri chiziqning tenglamasini $x = \xi(y)$ ko‘rinishida izlaymiz.

Agar egri chiziqning yoy elementini *ds* bilan belgilasak,

$$ds = \sqrt{1 + [\xi'(y)]^2}dy \qquad (1)$$

bo‘ladi.

h balandlikdan tushayotgan nuqtaning massasini *m* bilan, tezligini *v* bilan belgilasak, moddiy nuqta boshlang‘ich *M* nuqtadan ixtiyoriy *N* nuqtagacha harakatlanganda uning kinetik ener giyasining o‘zgarishi og‘irlik kuchining bajargan ishiga teng bo‘ladi ya’ni

$$\frac{1}{2}mv^2 = mg(h - y)$$

$$\frac{1}{2}v^2 = g(h - y) \qquad (2)$$

Tenglikka ega bo‘lamiz, bunda *g* – erkin tushish tezlanishi, *mg* esa og‘irlik kuchidir. Agar $v = ds / dt$ ekanini nazarga olsak, oxirgi tenglikdan quydagiga ega bo‘lamiz:

$$\frac{1}{2}\left(\frac{ds}{dt}\right)^2 = g(h - y).$$

$$ds = \sqrt{1 + [\xi'(y)]^2}dy \qquad (3)$$

(1), (2), (3) dan quyidagi tenglik hosil bo‘ladi.

$$dt = \frac{-\,ds}{\sqrt{2g(h - y)}} = \frac{1}{\sqrt{2g}}\frac{-\sqrt{1 + [\xi'(y)]^2}}{\sqrt{h - y}}dy. \qquad (4)$$

(Bu tenglikda *t* o‘sishi bilan nuqtaning ordinatasi *y* ning kamayishi minus ishora bilan hisobga olingan.)

M nuqtadan *O* nuqtaga tushish (o‘tish) vaqti *y* ning *h* dan *O* gacha o‘zgarishiga mos kelganligi sababli oxirgi tenglikdan quydagiga ega bo‘lamiz:

$$\psi(h) = T = \frac{1}{\sqrt{2g}}\int_0^h \frac{\sqrt{1+[\xi'(y)]^2}}{\sqrt{h-y}}dy. \qquad (5)$$

Bu yerda

$$\varphi(y) = \frac{\sqrt{1+[\xi'(y)]^2}}{\sqrt{2g}} \qquad (6)$$

deb belgilasak, masala ushbu

$$\int_0^h \frac{\varphi(y)}{\sqrt{h-y}}dy = \psi(h) \qquad (7)$$

(7) tenglama birinchi tur chiziqli integral tenglamaning yechimini topishga keltiriladi. Bu tenglama Abel nomi bilan yuritiladi.

II. Koshi masalasi.

Ushbu

$$\frac{d^n y}{dt^n} + a_1(t)\frac{d^{n-1}y}{dt^{n-1}} + \dots + a_n(t)y = \psi(t) \qquad (8)$$

n– tartibli chiziqli differinsial tenglamaning

$$y(t_0) = A_0, \quad \frac{dy(t_0)}{dt} = A_1, \cdots, \frac{d^{n-1}y(t_0)}{dt^{n-1}} = A_{n-1} \qquad (9)$$

boshlang‘ich shartlarini qanoatlantiruvchi yechimini toping, bu yerda $\psi(t)$ va $a_i(t)$ $(i = \overline{1,n})$ lar $a \le t \le b$ segmentda uzluksiz funksiyalar bo‘lib, $A_j(j = \overline{0, n-1})$ lar esa o‘zgarmas sonlardir.

Bu masalani yechish ikkinchi tur Vol’terra integral tenglamasining yechimini topish masalasiga keltirilishi mumkin. Yaqqollik uchun biz tegishli hisoblashlarni $n = 2$ bo‘lganda amalga oshiramiz.

Ushbu

$$\frac{d^2y}{dt^2} + a(t)\frac{dy}{dt} + b(t)y = \psi(t) \qquad (10)$$

Tenglama va

$$y(a) = A, \qquad \frac{dy(a)}{dt} = B \qquad (11)$$

Boshlang‘ich shartlar berilgan bo‘lsin. Bu masalaga mos integral tenglama tuzish maqsadida

$$\frac{d^2y}{dt^2} = \varphi(t) \qquad (*)$$

Belgilash kiritamiz. Agar $\varphi(t)$ ma’lum funksiya bo‘lsa (*) ni ketma – ket ikki marta integrallab va (11) ni hisobga olib (10) differensial tenglamaning tegishli yechimini toppish mumkin. Endi (*) ni integrallab topamiz:

$$\frac{dy}{dt} = \int_a^t \varphi(s)ds + C_1.$$

(11) shartdan foydalanib, C_1 ni aniqlaymiz:

$$B = \frac{dy(a)}{dt} = \int_a^a \varphi(s)ds + C_1 = C_1.$$

Ya’ni $C_1 = B$.

Shuning uchun quydagiga egamiz:

$$\frac{dy}{dt} = \int_a^t \varphi(s)ds + B.$$

Bu tenglikning ikkala tomonini integrallab topmiz:

$$y(t) = \int_a^t \left(B + \int_a^s \varphi(\tau)d\tau\right)ds + C_2 = \int_a^\tau ds \int_a^s \varphi(\tau)d\tau + B(t-a) + C_2.$$

Bundan $t = a$ bo‘lganda (11) ga ko‘ra

$$A = y(a) = C_2$$

Kelib chiqadi. Shunga asosan,

$$y(t) = \int_a^t ds \int_a^s \varphi(\tau)d\tau + B(t-a) + A.$$

Ravshanki,

$$\int_a^t ds \int_a^s \varphi(\tau)d\tau = \int_a^t \varphi(\tau) \int_\tau^t dsd\tau = \int_a^t (t-\tau)\varphi(\tau)d\tau.$$

Demak,

$$y(t) = \int_{a}^{t} (t - s)\varphi(s)ds + B(t - a) + A.$$

d^2y/dt^2, $dy\ /\ dt$ va u lar uchun topilgan ifodalarni (10) tenglamaga qo'yib, uni quydagi Vol'terra tenglamasiga keltiramiz:

$$\varphi(t) = f(t) + \int_{a}^{t} K(t, s)\ \ \varphi(s)ds,$$

Bunda

$$f(t) = -[Ba(t) + B(t - a)b(t) + Ab(t)] + \psi(t),$$
$$K(t, s) = -a(t) - (t - s)b(t).$$

Demak, (10) tenglama uchun koshi masalasi ikkinchi tur chiziqli Vol'terra tenglamasini yechish masalasiga keltiriladi.

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