# GAMMA FUNKSIYA

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## *ANNOTATSIYA*

*Beta va Gamma funksiyalar kasr tartibli integral tenglamalarni va hosilalarni hisoblashda foydalaniladi. Mittag – Leffler funksiyasi esa Riman-Liuvill ma’nosidagi kasr tartibli differensial tenglamalarni hisoblashda bizga kerak bo‘ladi.*

***Kalit so‘zlar:*** *Betta va Gamma funksiya*, *Kasr tartibli hosila, Xosmas integral, Veyershtass alomati, Eyler integrali.*

Biz

$$\int_0^{+\infty} x^{a-1}e^{-x}dx \qquad (6)$$

xosmas integralni qaraylik.Bu chegaralanmagan funksiyaning ($a<1$ da $x=0$ maxsus nuqta) cheksiz oraliq bo‘yicha olingan xosmas integrali bo‘lishi bilan birga a ga (parametrga) ham bog‘liqdir.Xosmas integralning $a<1$ da $(0;+\infty)$ yaqinlashuvchi, $a\le 0$ da, ya’ni $\left[-\infty;0\right]$ da uzoqlashuvchi bo‘lishi ko‘rsatildi.

**1-ta’rif:** (6) integral gamma funksiya yoki ikkinchi tur Eyler integrali deb ataladi va $\Gamma(a)$ kabi belgilanadi.Demak,

$$\Gamma(a)=\int_0^{+\infty} x^{a-1}e^{-x}dx \qquad (1.7)$$

SHunday qilib, $\Gamma(a)$ funksiya $(0;+\infty)$ da berilgandir.Endi $\Gamma(a)$ funksiyaning xossalarini o‘rganaylik.

$1^0$ (6) integral $\Gamma(a)=\int\limits_0^{+\infty} x^{a-1}e^{-x}dx$ ixtiyoriy $[a_0,b_0]$ $(0<a_0<b_0<+\infty)$ oraliqda tekis yaqinlashuvchi bo‘ladi.

**Isbot:** (6) integralni quyidagi 2 qismga ajratib,

$$\int\limits_0^{+\infty} x^{a-1}e^{-x}dx=\int\limits_0^{1} x^{a-1}e^{-x}dx+\int\limits_1^{+\infty} x^{a-1}e^{-x}dx$$

Ularning har birini alohida-alohida tekis yaqnlashuvchilikka tekshiramiz.Agar $a_0(a_0>0)$ sonni olib, parameter a ning $a\geq a_0$ qiymatlari qaralsa, unda barcha $x\in(0;1]$ uchun

$$x^{a-1}e^{-x}\leq\frac{1}{x^{1-a_0}}$$

bo‘lib, ushbu Veyrshtrass alomatiga ko‘ra

$$\int\limits_0^{1} x^{a-1}e^{-x}dx$$

integral tekis yaqinlashuvchi bo‘ladi.Agar $b_0(b_0>0)$ sonni olib,parameter a ning $a\leq b_0$ qiymatlari qaraladigan bo‘lsa, unda barcha $x\geq 1$ uchun

$$x^{a-1}e^{-x}\leq x^{b_0-1}e^{-x}\leq(\frac{b_0+1}{e})^{b_0+1}\cdot\frac{1}{x^2}$$

bo‘lib,

$$\int\limits_1^{+\infty}\frac{1}{x^2}dx$$

Integralning yaqinlashuvchiligidan, yana Veyershtass alomatiga ko‘ra

$$\int\limits_1^{+\infty} x^{a-1}e^{-x}dx$$

integralning tekis yaqinlashuvchiligini bo;lishini topamiz.SHunday qilib,

$$\Gamma(a)=\int_{0}^{+\infty} x^{a-1}e^{-x}dx$$

$\left[a_0,b_0\right]$ $(0<a_0<b_0<+\infty)$ da tekis yaqinlashuvchi bo‘ladi.

$2^0$ . $\Gamma(a)$ funksiya $\left(0;+\infty\right)$ da uzluksiz hamda barcha tartibdagi uzluksiz hosilalarga ega va

$$\Gamma^{(n)}(a)=\int_{0}^{+\infty} x^{a-1}e^{-x}(\ln x)^n dx \qquad (n=1,2,...)$$

**Isbot:** $\forall a\in(0;+\infty)$ nuqtani olaylik.Unda shunday $\left[a_0,b_0\right]$ $(0<a_0<b_0<+\infty)$ oraliq topiladiki, $a\in\left[a_0;b_0\right]$ bo‘ladi.Ravshanki,

$$\Gamma(a)=\int_{0}^{+\infty} x^{a-1}e^{-x}dx$$

integral ostidagi $f(x,a)=x^{a-1}e^{-x}$ funksiya

$$M=\left\{\left(x,a\right)\in R^2 : x\in\left(0;+\infty\right), a\in\left(0;+\infty\right)\right\}$$

to‘plamda uzluksiz funksiyadir.(6) integral esa $\left[a_0;b_0\right]$ da tekis yaqinlashuvchi.

U holda teoremaga asosan $\Gamma(a)$ funksiya $\left[a_0;b_0\right]$ da binobarin, a nuqtada uzluksiz bo‘ladi. (6) integral ostidagi $f(x,a)=x^{a-1}e^{-x}$ funksiya

$$f_a^{'}(x,a)=x^{a-1}e^{-x}\ln x$$

hosilasining M to‘plamda uzluksiz funksiya.

Endi

$$\int_{0}^{+\infty} f_a^{'}(x,a)dx=\int_{0}^{+\infty} x^{a-1}e^{-x}\ln x dx$$

integralni $\left[a_0;b_0\right]$ da tekis yaqinlashuvchi bo‘lishini ko‘rsatamiz.Ushbu

$$\int_{0}^{1} x^{a-1}e^{-x}\ln x dx$$

integral ostidagi $x^{a-1}e^{-x}\ln x$ funksiya uchun

$$0 < x \le 1 \ \text{da} \quad \int_0^1 x^{\frac{a}{2}-1}dx$$

o‘rinlidir. $\Psi_1(x) = x^{\frac{a_0}{2}}\left|\ln x\right|$ funksiya $0 < x \le 1$ da chegaralanganligidan va $\int_0^1 x^{\frac{a}{2}-1}dx$ integralning yaqinlashuvchiligidan

ning ham yaqinlashuvchi bo‘lishini va Veyrshtrass alomatiga ko‘ra qaralayotgan

$\int_0^1 x^{a-1}e^{-x}\ln xdx$ integralning tekis yaqinlashuvchiligini topamiz.

SHunga o‘xshash quyidagi

$$\int_1^{+\infty} x^{a-1}e^{-x}\ln xdx$$

integralda,integral ostidagi $x^{a-1}e^{-x}\ln x$ funksiya uchun barcha $x \ge 1$ da

$$x^{a-1}e^{-x}\ln x \le x^{b_0-1}e^{-x}\ln x \le (\frac{b_0+2}{e})^{b_0+2}\frac{1}{x^2}$$

bo‘lib, $\int_1^{+\infty}\frac{dx}{x^2}$ integralning yaqinlashuvchiligidan, ya’na Veyrshtrass alomatiga ko‘ra $\int_1^{+\infty} x^{a-1}e^{-x}\ln xdx$ ning tekis yaqinlashuvchiligi kelib chiqadi.Demak, $\left[a_0;b_0\right]$ da $\int_1^{+\infty} x^{a-1}e^{-x}\ln xdx$ integral tekis yaqinlashuvchi.Unda teoremaga asosan

$$f^{'}(a) = (\int_0^{+\infty} x^{a-1}e^{-x}dx)^{'} = \int_0^{+\infty}(x^{a-1}e^{-x})^{'}dx = \int_0^{+\infty} x^{a-1}e^{-x}\ln xdx$$

bo‘ladi va $\Gamma'(a)$ $[a_0;b_0]$da binobarin, a nuqtada uzluksizdir.Xuddi shu yo‘l bilan $\Gamma(a)$ funksiyaning ikkinchi, uchinchi va hokazo tartibdagi hosilalarining mavjudligi,uzlukisizligi hamda $\Gamma^{(n)}(a)=\int_0^{+\infty} x^{a-1}e^{-x}\ln^n x dx$ $(n=1,2,...$ bo‘lishi ko‘rsatiladi)

$3^0.\Gamma(a)$ funksiya uchun ushbu $\Gamma(a+1)=a\cdot\Gamma(a)$ $(a>0)$ formula o‘rinli.

Haqiqatan ham,

$$\Gamma(a)=\int_0^{+\infty} x^{a-1}e^{-x}dx=\int_0^{+\infty} e^{-x}d(\frac{x^a}{a})$$

integralni bo‘laklab integrallasak,

$$\Gamma(a)=e^{-x}\cdot\frac{x^a}{a}\Big|_0^{+\infty}+\int_0^{+\infty}\frac{x^a}{a}e^{-x}dx=\frac{1}{a}\Gamma(a+1)$$

bo‘lib, undan

$$\Gamma(a+1)=a\,\Gamma(a) \qquad (7)$$

bo‘lishi kelib chiqadi.Bu formula yordamida $\Gamma(a+n)$ ni topish mumkin.

Darhaqiqat, (7) formulani takror qo‘llab

$$\Gamma(a+2)=\Gamma(a+1)(a+1)$$
$$\Gamma(a+3)=\Gamma(a+2)(a+2)$$
$$......................................$$
$$\Gamma(a+n)=\Gamma(a+n-1)(a+n-1)$$

bo‘lishini,ulardan esa

$$\Gamma(a+n)=(a+n-1)(a+n-2)...(a+2)(a+1)a\Gamma(a)$$

ekanligini topamiz.Xususan , $a=1$ bo‘lganda

$$\Gamma(n+1)=n(n-1)...2\cdot1\cdot\Gamma(1)$$

bo‘ladi.Agar

$$\Gamma(1)=\int_0^{+\infty} e^{-x}dx=1$$

bo‘lishini e’tiborga olsak,unda $\Gamma(n+1)=n!$ ekanligi kelib chiqadi.Yana, (7) formuladan foydalanib $\Gamma(2)=\Gamma(1)=1$ bo‘lishini topamiz.

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