KASR TARTIBLI ODDIY DIFFERENSIAL TENGLAMALAR

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ANNOTATSIYA

Kasr tartibli tenglamalarning fizik, texnik va biologik jaroyonlarga tadbiqi katta bo'lgani uchun butun dunyo olimlari tomonidan kasr tartibli tenglamalarni o'rganishga bo'lgan qiziqish ortib bormoqda. Bugungi kunda kasr tartibli aralash tipdagi tenglamalar uchun to'g'ri va teskari masalalarni o'rganish va yechish dolzarb masalaga aylandi.

Kalit so'zlar: *Kasr tartibli hosila, Riemann-Liouville, Koshiy masalasi, Volterra integral, Mittag-Leffler funksiyasi.*

Riemann-Liouville ma'nosidagi kasr tartibli differensial tenglamalar uchun Koshi tipidagi masalalar.

Ushbu kichik bo'limda biz chiziqli kasr tartibli differensial tenglamaning aniq echimlarini tuzamiz. Riman-Liouvil kasr tartibli hosilasi

$$D^{\alpha} f = \frac{1}{\Gamma(n-\alpha)}\left(\frac{d}{dx}\right)^{n}\int_{0}^{x}\frac{f(t)}{(x-t)^{\alpha-n+1}}dt.$$

Bilan aniqlanga quyidagi boshlang'ich shartli Koshi masalasini qaraylik:

$$\begin{aligned}&\left(D^{\alpha} y\right)(x)-\lambda y(x)=f(x),\left(a<x\leq b,\alpha>0,\lambda\in R\right)\\&\left(D^{\alpha-k} y\right)(a+)=b_k,\left(b_k\in R,k=1,2,...n=-[-\alpha]\right)\end{aligned} \qquad (1),(2)$$

Faraz qilaylik, $f(x)\in C_{\gamma}[a,b],(0\le\gamma<1)$ bo‘lsin. U holda , $C_{n-\alpha}[a,b]$ fazoda (1),(2) koshi masalasi quyidagi

$$y(x)=\sum_{j=1}^{n}\frac{b_j}{\Gamma(\alpha-j+1)}(x-a)^{\alpha-j}+\frac{\lambda}{\Gamma(\alpha)}\int_a^x\frac{y(t)}{(x-t)^{1-\alpha}}dt+\frac{1}{\Gamma(\alpha)}\int_a^x\frac{f(t)}{(x-t)^{1-\alpha}}dt \quad (3)$$

Volterra integral tenglamasiga ekvivalent tenglama bo‘ladi.

Biz bu tenglamani ketma ket yaqinlashish metodi orqali yechimini topamiz.

Bu metodga ko‘ra,

$$y_0(x)=\sum_{j=1}^{n}\frac{b_j}{\Gamma(\alpha-j+1)}(x-a)^{\alpha-j}$$
$$y_m(x)=y_0(x)+\frac{\lambda}{\Gamma(\alpha)}\int_a^x\frac{y_{m-1}(t)}{(x-t)^{1-\alpha}}dt+\frac{1}{\Gamma(\alpha)}\int_a^x\frac{f(t)}{(x-t)^{1-\alpha}}dt \quad (4),(5)$$

Bu yaqinlashishlarni operator ko‘rinishida quyidagicha yozib olishimiz mumkin:

$$y_0(x)=\sum_{j=1}^{n}\frac{b_j}{\Gamma(\alpha-j+1)}(x-a)^{\alpha-j}$$
$$y_m(x)=y_0(x)+\lambda\left(I^{\alpha}y_{m-1}\right)(x)+\left(I^{\alpha}f\right)(x) \quad (4),(6)$$

Yuqoridagi formula orqali y_1 ni hisoblab olamiz,

$$y_1(x)=y_0(x)+\lambda\left(I^{\alpha}y_0\right)(x)+\left(I^{\alpha}f\right)(x)=$$

$$=\sum_{j=1}^{n}\frac{b_j}{\Gamma(\alpha-j+1)}(x-a)^{\alpha-j}+\lambda\sum_{j=1}^{n}\frac{b_j}{\Gamma(2\alpha-j+1)}(x-a)^{2\alpha-j}+\left(I^{\alpha}f\right)(x)=$$

$$=\sum_{j=1}^{n}b_j\sum_{k=1}^{2}\frac{\lambda^{k-1}(x-a)^{\alpha k-j}}{\Gamma(\alpha k-j+1)}+\frac{1}{\Gamma(\alpha)}\int_a^x(x-t)^{\alpha-1}f(t)dt\cdot$$

Yuqoridagiga o‘xshash y_2 uchun ham quyidagi formulani yozib olamiz:

$$y_2(x) = y_0(x) + \lambda\left(I^{\alpha} y_1\right)(x) + \left(I^{\alpha} f\right)(x)$$

Bundan,

$$y_2(x) = y_0(x) + \lambda\left(I^{\alpha} y_1\right)(x) + \left(I^{\alpha} f\right)(x) = \sum_{j=1}^{n} \frac{b_j}{\Gamma(\alpha - j + 1)}(x-a)^{\alpha - j} +$$

$$+\sum_{j=1}^{n} b_j \sum_{k=1}^{2} \frac{\lambda^{k-1}}{\Gamma(\alpha k - j + 1)}\left(I^{\alpha}(t-a)^{\alpha k - j}\right)(x) + \left(I^{\alpha} f\right)(x) + \left(I^{\alpha} I^{\alpha} f\right)(x)$$

Buni quyidagicha ixchamlab yozish mumkin:

$$y_2 = \sum_{j=1}^{n} b_j \sum_{k=1}^{3} \frac{\lambda^{k-1}(x-a)^{\alpha k - j}}{\Gamma(\alpha k - j + 1)} + \int_a^x \left[\sum_{k=1}^{2} \frac{\lambda^{k-1}}{\Gamma(\alpha k)}(x-t)^{\alpha k - 1}\right] f(t)\,dt \qquad (7)$$

Jarayonni davom ettirib quyidagi ketma-ketlikni hosil qilamiz:

$$y_m = \sum_{j=1}^{n} b_j \sum_{k=1}^{m+1} \frac{\lambda^{k-1}(x-a)^{\alpha k - j}}{\Gamma(\alpha k - j + 1)} + \int_a^x \left[\sum_{k=1}^{m} \frac{\lambda^{k-1}}{\Gamma(\alpha k)}(x-t)^{\alpha k - 1}\right] f(t)\,dt \qquad (8)$$

Bu ketma-ketlikda m ni cheksizga intiltirib limitga o‘tsak (3) integral tenglamaning yechimiga kelamiz.

$$y(x) = \sum_{j=1}^{n} b_j \sum_{k=1}^{\infty} \frac{\lambda^{k-1}(x-a)^{\alpha k - j}}{\Gamma(\alpha k - j + 1)} + \int_a^x \left[\sum_{k=1}^{\infty} \frac{\lambda^{k-1}}{\Gamma(\alpha k)}(x-t)^{\alpha k - 1}\right] f(t)\,dt \qquad (9)$$

Endi bu yechimni ko‘rinishini ixchamlash uchun 1-bobda keltirib o‘tilgan

$E_{\alpha,\beta}(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(\alpha k + \beta)}$ Mittag-Leffler funksiyasidan foydalanamiz.

Kerakli o‘rinlarda bu funksiya ifodasini almashtirsak quyidagi ko‘rinishdagi yechimga ega bo‘lamiz:

$$y(x) = \sum_{j=1}^{n} b_j (x-a)^{\alpha - j} E_{\alpha,\alpha - j + 1}\left[\lambda(x-a)^{\alpha}\right] + \int_a^x (x-t)^{\alpha - 1} E_{\alpha,\alpha}\left[\lambda(x-a)^{\alpha}\right] f(t)\,dt$$

(10)

Bu funksiya (3) Volterra integral tenglamasining yechimi bo‘ladi va demak (1),(2) Koshi masalasining ham yechimini ifodalaydi

Agar Koshi masalasida $0<\alpha<1, \lambda \in R$ bo‘lsa,

$$\left(D^{\alpha} y\right)(x)-\lambda y(x)=f(x)$$

$$\left(D^{\alpha-1} y\right)(a+)=b$$

yechim quyidagicha bo‘ladi:

$$y(x)=b(x-a)^{\alpha} E_{\alpha,\alpha}\left[\lambda(x-a)^{\alpha}\right]+\int_{a}^{x}(x-t)^{\alpha-1} E_{\alpha,\alpha}\left[\lambda(x-a)^{\alpha}\right] f(t) dt \cdot$$

Agar ozod had nolga teng bo‘lsa,

$$\left(D^{\alpha} y\right)(x)-\lambda y(x)=0$$

$$\left(D^{\alpha-1} y\right)(a+)=b$$

Quyidagicha soda ko‘rinishdagi yechim olinadi:

$$y(x)=b(x-a)^{\alpha} E_{\alpha,\alpha}\left[\lambda(x-a)^{\alpha}\right].$$

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