CAPUTO MA'NOSIDAGI KASR TARTIBLI TENGLAMALARDA MANBA FUNKSIYANI ANIQLASH BO'YICHA TO'G'RI MASALALAR

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ANNOTATSIYA

Bu maqolada Kaputo ma'nosida kasr tartibli xususiy hosilali differensial tenglama uchun aralash masalani o'rganamiz. Ushbu maqolada to'g'ri masalani yechish va Caputo ma'nosida kasr tartibli xususiy hosilali differensial tenglamaning yechimi mavjud va yagonaligini ko'rsatish, hamda manba funksiyasini aniqlash bo'yicha to'g'ri masalaga oid natijalar olinishi ko'zda tutilgan.

Kalit so'zlar: *Kaputo hosilali differensial tenglama, to'g'ri masala, teskari masala, Koshi masalasi, fiksirlangan son.*

Aytaylik, $0<\rho<1$ bo'lsin. Biz quyidagi

$$D_t^{\rho}u(x,t)-a^2u_{xx}(x,t)=f(x),\qquad 0<x<l,\quad 0<t<T;\qquad (1.1)$$

Kaputo ma'nosida kasr tartibli tenglamaning

$$u(x,+0)=\varphi(x),\qquad 0\le x\le l,\qquad (1.2)$$

boshlang'ich shartni va quyidagi

$$u(0,t)=0,\qquad 0\le t\le T,\qquad (1.3)$$

$$u(l,t)=0,\qquad 0\le t\le T,\qquad (1.4)$$

chegaraviy shartlarni qanoatlantiruvchi yechimini topish masalasini qaraylik, bu yerda $\varphi(x)$, $f(x)$ – berilgan funksiyalar, a – o'zgarmas son, T – fiksirlangan son, D_t^{ρ} orqali Caputo ma'nosidagi ρ- tartibli kasr tartibli hosila belgilangan.

(1.1) – (1.4) masalaning yechimini topish masalasiga ***to‘g‘ri masala*** deyiladi.

3.1.1-ta’rif. Agar $u(x,t)\in C\left([0,l]\times[0,T]\right)$ funksiya quyidagi $D_t^{\rho}u(x,t)$, $u_{xx}(x,t)\in C\left((0,l)\times(0,T\right)$ xossaga ega bo‘lib, (1.1) - (1.4) ning barcha shartlarini qanoatlantirsa, u holda bu $u(x,t)$ funksiyaga (1.1) - (1.4) ***masalaning yechimi*** deb ataladi.

Magistrlik dissertatsiyasida ushbu to‘g‘ri masalaning yechimini topish bilan

birga manba funksiyasini topish bo‘yicha teskari masala ham o‘rganilgan.

Faraz qilaylik (1.1) – (1.4) masalada $u(x,t)$ funksiyadan tashqari $f(x)$ funksiya ham noma’lum bo‘lsin. Bu masalani yechish uchun bizga qo‘shimcha shart kerak boladi. Biz qo‘shimcha shart sifatida quyidagi shartni olamiz:

$$u(x,\tau)=\psi(x), \qquad 0<\tau<T. \qquad (1.5)$$

Ushbu (1.1) – (1.5) masalada $u(x,t)$ va $f(x)$ funksiyalarni topish masalasiga tenglamaning o‘ng tomonini topish bo‘yicha ***teskari masala*** deb ataladi.

To‘g‘ri masalani yechish

Kasr tartibli xususiy hosilali differensial tenglama uchun to‘g‘ri masalasini yechish ko‘rsatilgan, ya’ni (1.1) – (1.4) to‘g‘ri masalaning yechimi mavjud va yagonaligi isbotlangan.

(1.1) – (1.4) masalani yechish uchun quyidagi teoremani isbotlaymiz.

1.1-teorema. $\varphi(x)$, $f(x)$ funksiyalar uzluksiz, bo‘lakli - uzluksiz hosilaga ega va $\varphi(0)=\varphi(l)=0$, $f(0)=f(l)=0$ shartlarni qanoatlantiruvchi funksiyalar bo‘lsin. U holda (1.1) - (1.4) masalaning yechimi yagona bo‘ladi va u quyidagicha ko‘rinishiga ega:

$$u(x,t)=\sum_{n=1}^{\infty}\left[\varphi_n E_{\rho,1}\left(-\left(\frac{\pi na}{l}\right)^2 t^{\rho}\right)+\right.$$

$$\left.+f_n t^{\rho}E_{\rho,\rho+1}\left(-\left(\frac{\pi na}{l}\right)^2 t^{\rho}\right)\right]\sin\frac{\pi nx}{l}. \qquad (1.6)$$

Isbot. Teoremani isbotlash uchun xususiy hosilali tenglamalarni yechishda keng tarqalgan usullardan biri o‘zgaruvchilarni ajratish, ya’ni Furye (Fourier) usulidan foydalanamiz. (1.1) – (1.4) masalaning yechimini

$$u(x,t) = v(x,t) + w(x,t)$$

ko‘rinishda izlaymiz, bu yerda $v(x,t)$ funksiya

$$D_t^{\rho} v(x,t) - a^2 v_{xx}(x,t) = 0, \qquad 0 < x < l, \quad 0 < t < T; \qquad (1.7)$$

$$v(x,+0) = \varphi(x), \qquad 0 \le x \le l, \qquad (1.8)$$

$$v(0,t) = 0, \qquad 0 \le t \le T, \qquad (1.9)$$

$$v(l,t) = 0, \qquad 0 \le t \le T. \qquad (1.10)$$

masalaning, $w(x,t)$ funksiya esa

$$D_t^{\rho} w(x,t) - a^2 w_{xx}(x,t) = f(x), \qquad 0 < x < l, \quad 0 < t < T; \qquad (1.11)$$

$$w(x,+0) = 0, \qquad 0 \le x \le l, \qquad (1.12)$$

$$w(0,t) = 0, \qquad 0 \le t \le T, \qquad (1.13)$$

$$w(l,t) = 0, \qquad 0 \le t \le T. \qquad (1.14)$$

masalaning yechimi.

(1.1) – (1.4) masalani yechish uchun yuqoridagi ikkita yordamchi masalalarni yechish kifoya qiladi.

Yuqorida qaraganimizdek ushbu qismda (1.1) – (1.4) masalani bir jinsli va bir jinsli bo‘lmagan ikki hol uchun alohida-alohida yechib olamiz.

(1.7) – (1.10) masalani yechish uchun Furye usulidan foydalanamiz. Yechimni

$$v(x,t) = T(t) \cdot X(x) \neq 0 \qquad (1.15)$$

ko‘rinishda izlaymiz, bunda $X(x)$ – faqat x o‘zgaruvchining funksiyasi, $T(t)$ – esa faqat t o‘zgaruvchining funksiyasidir.

Endi esa, $\lambda_n = \left(\frac{\pi n}{l}\right)^2$ xos qiymatga mos xos funksiyani $T_n(t)$ lar uchun quyidagi ifodalarni topamiz:

$$v(x,t)=\sum_{n=1}^{\infty}T_n(t)\cdot\sin\frac{\pi nx}{l} \qquad (1.24)$$

(1.24) ifodani (1.7) masalaga olib borib qo‘ysak, quyidagi tenglik hosil bo‘ladi:

$$\sum_{n=1}^{\infty}D_t^{\rho}T_n(t)\cdot\sin\frac{\pi nx}{l}+a^2\sum_{n=1}^{\infty}\left(\frac{\pi n}{l}\right)^2T_n(t)\cdot\sin\frac{\pi nx}{l}=0.$$

Bundan esa,

$$\sum_{n=1}^{\infty}\left[D_t^{\rho}T_n(t)+a^2\left(\frac{\pi n}{l}\right)^2T_n(t)\right]\cdot\sin\frac{\pi nx}{l}=0$$

tenglikni hosil qilamiz. Shunday qilib biz quyidagi masalaga kelamiz:

$$\begin{cases}D_t^{\rho}T_n(t)+a^2\left(\dfrac{\pi n}{l}\right)^2T_n(t)=0,\\ T_n(+0)=\varphi_n,\end{cases} \qquad (1.25)$$

(3.1.25) Koshi masalasining yechimi (2.2.14) ga asosan quyidagicha bo‘ladi (qarang [Kilbas]):

$$T_n(t)=\varphi_nE_{\rho,1}\left(-\left(\frac{\pi na}{l}\right)^2t^{\rho}\right). \qquad (1.26)$$

Xususiy yechimlar yig‘indisi ya’na yechim bo‘lgani uchun

$$v(x,t)=\sum_{n=1}^{\infty}T_n(t)\cdot X_n(t)$$

funksiya ham yechim bo‘ladi. Shunday qilib (1.7) – (1.10) masalaning formal yechimi

$$v(x,t)=\sum_{n=1}^{\infty}\varphi_nE_{\rho,1}\left(-\left(\frac{\pi na}{l}\right)^2t^{\rho}\right)\sin\frac{\pi nx}{l} \qquad (1.27)$$

ko‘rinishda bo‘ladi.

Bundan tashqari, (1.11) tenglik $D_t^{\rho}W_j(x,t)=\dfrac{\partial^2}{\partial x^2}W_j(x,t)+\sum_{k=1}^{j}f_k(t)\sin\dfrac{\pi nx}{l}$, $t>0$ dan $D_t^{\rho}w(x,t)\in C((0,l)\times(0,T))$ ekanligi kelib chiqadi. Demak, yuqoridagi mulohazalardan (1.30) funksiya (1.11) – (1.14) masalaning yechimi ekanligi kelib chiqadi.

Bu yechimlarni umumlashtirib (1.7) – (1.10) masala uchun quyidagi yechimga ega bo‘lamiz:

$$u(x,t)=v(x,t)+w(x,t)=\sum_{n=1}^{\infty}\varphi_n E_{\rho,1}\left(-\left(\frac{\pi na}{l}\right)^2 t^{\rho}\right)\sin\frac{\pi nx}{l}+$$

$$+\sum_{n=1}^{\infty} f_n t^{\rho} E_{\rho,\rho+1}\left(-\left(\frac{\pi na}{l}\right)^2 t^{\rho}\right)\cdot\sin\frac{\pi nx}{l}. \qquad (1.31)$$

Shunday qilib, (1.6) formula bilan aniqlangan $u(x,t)$ funksiya (1.7) – (1.10) masalaning yechimi bo‘lar ekan.

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