IKKINCHI VA UCHINCHI TIP KLASSIK SOHALAR DEKART KO‘PAYTMALARINING BERGMAN YADROSI

Fayzullayev Shoxzodbek Maqsud o‘g‘li
Urganch Ranch Texnologiya Universiteti
fayzshoxzod@gmail.com

ANNOTATSIYA

Ushbu maqolada bir jinsli doiraviy matritsaviy sohalarning Bergman yadrolari topish va ularning xossalari o‘rganilgan.

Kalit so‘zlar. *Klassik sohalar, Bergman yadrosi, yakobiyan, doiraviy soha, avtomorfizm.*

ANNOTATION

This work is devoted to the search for Bergman kernels of homogeneous circular matrix domains and their properties..

Keywords. *Classical domains, Bergman kernel, Jacobian, circular domain, automorphism.*

Ma’lumki,

$$\Phi_2 = G(B - P_2)\left(I^{(n)} - \overline{P}_2 B\right)^{-1}\overline{G}^{-1}, \quad (1.1)$$

akslantirish esa, $\Re_{II}(n)$ ikkinchi tip klassik sohaning, P_2 nuqtasini koordinata boshiga akslantiruvchi avtomorfizmi bo’ladi, bu yerda $Z \in \Re_{II}(n)$ va $G \in \mathbb{C}[n \times n]$ matritsalar quyidagi shartlarni qanoatlantiradi:

$$\overline{G}\left(I^{(n)} - \overline{P_2}P_2'\right)G' = I^{(n)}. \quad (1.2)$$

Shuningdek, quyidagi

$$\Phi_3 = A(Z - P_3)\left(I^{(n)} + \overline{P}Z\right)^{-1}\bar{A}^{-1} \quad (1.3)$$

akslantirish esa, $\Re_{III}(n)$ ikkinchi tip klassik sohaning, P_3 nuqtasini koordinata boshiga akslantiruvchi avtomorfizmi bo'ladi, bu yerda $Z \in \Re_{III}(n)$va $A \in \mathbb{C}[n \times n]$ matritsalar quyidagi shartlarni qanoatlantiradi:

$$\overline{A}\left(I^{(n)} + P_3\overline{P_3}\right)A' = I^{(n)}. \qquad (1.4)$$

Aytaylik, $\Re$ sohani $\Re_{II}(n)$ va $\Re_{III}(n)$ sohalarining dekart ko'paytmasi quyidagi ko'rinishida berilgan bo'lsin:

$$\Re = \Re_{II}(n) \times \Re_{III}(n) = \{(B, Z): B \in \Re_{II}(n), Z \in \Re_{III}(n)\},$$

bu yerda,

$$\Re_{II}(n) = \left\{B \in \mathbb{C}[n \times n]: I^{(n)} - B\overline{B} > 0, \forall B' = B\right\}.$$

va

$$\Re_{III}(n) = \left\{Z \in \mathbb{C}[n \times n]: I^{(n)} + Z\overline{Z} > 0, \forall Z' = -Z\right\}.$$

Bu $\Re$ sohaning $\mathbb{X}$ ostovi, $\Re_{II}(n)$ va$\Re_{III}(n)$ sohalar $\mathbb{X}_{II}$ va $\mathbb{X}_{III}$ ostovlari dekart ko'paytmasiga teng bo'ladi, ya'ni

$$\mathbb{X}_{II} = \left\{B \in \mathbb{C}[n \times n]: B\overline{B} = I^{(n)}, \forall B' = B\right\},$$

$$\mathbb{X}_{III} = \left\{Z \in \mathbb{C}[n \times n]: -Z\overline{Z} = I^{(n)}, \forall Z' = -Z\right\}$$

$$\mathbb{X} = \mathbb{X}_{II} \times \mathbb{X}_{III}.$$

U holda yuqoridagi (1.1) va (1.3) munosabatlarga asosan quyidagi tasdiq o'rinli bo'lishini topamiz:

1-tasdiq. *Komponentalari mos ravishda (1.2), (1.4) shartlar bilan aniqlanadigan, (1.1) hamda (1.3) ko'rinishidagi Φ_2 va Φ_3 akslantirishlar bilan aniqlanadigan ushbu $\Phi = (\Phi_2, \Phi_3)$ akslantirish $\Re = \Re_{II}(n) \times \Re_{III}(n)$ sohaning $P = (P_2, P_3) \in \Re$ nuqtasini koordinata boshiga akslantiruvchi avtomorfizmi bo'ladi.*

Bremerman teoremasini $\Re_{II}(n)$ va $\Re_{III}(n)$ klassik sohalar dekart ko'paytmasinining Bergman yadrosini topish uchun analogini keltiramiz.

1-teorema. *Aytaylik,* $\Re_{II}(n)$ *va* $\Re_{III}(n)$ *sohalar* $B \in \mathbb{C}[n \times n]$ *va* $Z \in \mathbb{C}[n \times n]$ *o'zgaruvchili fazolardagi klassik sohalar va* $\Re = \Re_{II}(n) \times \Re_{III}(n)$ *bo'lsin, u holda quyidagi tenglik o'rinli bo'ladi:*

$$K_{\Re}(B, Z, \overline{B}, \overline{Z}) = K_{\Re_{II}(n)}(B, \overline{B}) K_{\Re_{III}(n)}(Z, \overline{Z}) \tag{1.5}$$

Isbot. Buning uchun 1-tasdiqga asoslanib, $\Re$ sohaning $\Phi = (\Phi_2, \Phi_3)$ avtomorfizmi xususiyatlaridan foydalanamiz. Dastlab, (1.1) ko'rinishdagi Φ_2 akslantirishni differensiallab ushbu

$$d\Phi_2 = G\left[dB \cdot \left(I^{(n)} - \overline{P}_2 B\right)^{-1} + (B - P_2) d\left(I^{(n)} - \overline{P}_2 B\right)^{-1}\right] \overline{G}^{-1},$$

tenglikka ega bo'lamiz. Agar $B = P_2$ desak u holda (1.2) shartga asosan quyidagi tenglikka ega bo'lamiz:

$$d\Phi_2 = G \cdot dB \cdot \left(I^{(n)} - \overline{P}_2 P_2\right)^{-1} \cdot \overline{G}^{-1} = G \cdot dB \cdot G'. \tag{1.6}$$

va (1.5) munosabat ushbu

$$B = \begin{pmatrix} b_{11} & b_{12} & \dots & b_{1n} \\ b_{21} & b_{22} & \dots & b_{2n} \\ \dots & \dots & \dots & \dots \\ b_{n1} & b_{n2} & \dots & b_{nn} \end{pmatrix}, G = \begin{pmatrix} g_{11} & g_{12} & \dots & g_{1n} \\ g_{21} & g_{22} & \dots & g_{2n} \\ \dots & \dots & \dots & \dots \\ g_{n1} & g_{n2} & \dots & g_{nn} \end{pmatrix}, \Phi_2 = \begin{pmatrix} h_{11} & h_{12} & \dots & h_{1n} \\ h_{21} & h_{22} & \dots & h_{2n} \\ \dots & \dots & \dots & \dots \\ h_{n1} & h_{n2} & \dots & h_{nn} \end{pmatrix}.$$

belgilashlar orqali, quyidagi tenglikka ekvivalentdir:

$$dh_{sj} = \sum_{l=1}^{n} \sum_{i=1}^{n} g_{si} db_{il} g_{lj}, s = 1,2,..,n; j = 1,2,..,n.$$

Bu oxirgi tenglikdan $\Phi_2 = (h_{11}, \dots, h_{nn})$ golomorf akslantirish uchun quyidagi tenglik o'rinli bo'lishi kelib chiqadi:

$$dh_{11} \wedge dh_{12} \wedge \dots \wedge dh_{nn} = \prod_{s,j} \sum_{l=1}^{n} \sum_{i=1}^{n} g_{si} db_{il} g_{lj}, s = 1,2,..,n; j = 1,2,..,n.$$

U holda $db_{il} = db_{li}$ ekanidan, ushbu

$$dh_{11} \wedge dh_{12} \wedge \dots \wedge dh_{nn} =$$

$$= \prod_{s,j} (\det G)^{s} \cdot db_{11} \wedge db_{12} \dots \wedge db_{nn} \cdot (\det \overline{G})^{j}, s = 1,2,..,n; j = 1,2,..,n.$$

tenglik quyidagi

$$dh_{11} \wedge dh_{12} \wedge \dots \wedge dh_{nn} = (\det G)^{\frac{n+1}{2}} \cdot (\det \overline{G})^{\frac{n+1}{2}} \cdot db_{11} \wedge db_{12} \dots \wedge db_{nn} =$$

$$= J_{\mathbb{C}}(\Phi_2) \cdot db_{11} \wedge db_{12} \ldots \wedge db_{nn},$$

tenglikka ekvivalent bo'ladi, bu yerda $J_{\mathbb{C}}(\Phi_2) - \Phi_2$ akslantirishning kompleks yakobiani va u quyidagicha topiladi:

$$J_{\mathbb{C}}(\Phi_2) = (\det G)^{\frac{(n+1)}{2}} \times (\det G')^{\frac{(n+1)}{2}} = (\det G')^{(n+1)}.$$

Bulardan (1.2) shartga asosan

$$\dot{\Phi}_2 = \left|J_{\mathbb{C}}(\Phi_2)\right|^2 \dot{B} = \left|(\det G)^{n+1}\right|^2 \dot{B} = \frac{\dot{B}}{\det^{n+1}\left(I - B\overline{B}\right)}$$

tenglikka ega bo'lamiz.

Endi (1.3) ko'rinishdagi Φ_3 akslantirishni differensiallab ushbu

$$d\Phi_3 = A\left[dZ \cdot \left(I^{(n)} - \overline{P}_3 Z\right)^{-1} + (Z - P_3)d\left(I^{(n)} + \overline{P}_3 Z\right)^{-1}\right]\overline{A}^{-1},$$

tenglikka ega bo'lamiz. Agar $Z = P_2$ desak u holda (1.4) shartga asosan quyidagi tenglikka ega bo'lamiz:

$$d\Phi_3 = A \cdot dZ \cdot \left(I^{(n)} + \overline{P}_3 P_3\right)^{-1} \cdot \overline{A}^{-1} = A \cdot dZ \cdot A'. \qquad (1.6)$$

va (1.6) munosabat ushbu

$$Z = \begin{pmatrix} 0 & z_{12} & \ldots & z_{1n} \\ z_{21} & 0 & \ldots & z_{2n} \\ \ldots & \ldots & \ldots & \ldots \\ z_{n1} & z_{n2} & \ldots & 0 \end{pmatrix}, \qquad A = \begin{pmatrix} 0 & a_{12} & \ldots & a_{1n} \\ a_{21} & 0 & \ldots & a_{2n} \\ \ldots & \ldots & \ldots & \ldots \\ a_{n1} & a_{n2} & \ldots & 0 \end{pmatrix},$$

$$\Phi_3 = \begin{pmatrix} v_{11} & v_{12} & \ldots & v_{1n} \\ v_{21} & v_{22} & \ldots & v_{2n} \\ \ldots & \ldots & \ldots & \ldots \\ v_{n1} & v_{n2} & \ldots & v_{nn} \end{pmatrix}$$

belgilashlar orqali quyidagi tenglikka ekvivalentdir:

$$dv_{sj} = \sum_{l=1}^{n} \sum_{i=1}^{n} a_{si} dz_{il} a_{lj}, s = 1,2,..,n; j = 1,2,..,n.$$

Bu oxirgi tenglikdan $\Phi_3 = (v_{11},\dots,v_{nn})$golomorf akslantirish uchun quyidagi tenglik o'rinli bo'lishi kelib chiqadi:

$$dv_{11} \wedge dv_{12} \wedge\dots\wedge dv_{nn} = \prod_{s,j} \sum_{l=1}^{n} \sum_{i=1}^{n} a_{si} dz_{il} a_{lj}, s = 1,2,\dots,n; j = 1,2,\dots,n.$$

U holda $dz_{il} = -dz_{li}$ ekanidan, ushbu

$$dv_{11} \wedge dv_{12} \wedge\dots\wedge dv_{nn} =$$

$$= \sum_{s,j} (\det A)^{s} \cdot dz_{11} \wedge dz_{12} \dots \wedge dz_{nn} \cdot (\det A')^{j}, s = 1,2,\dots,n; j = 1,2,\dots,n.$$

tenglik quyidagi

$$dv_{11} \wedge dv_{12} \wedge \dots \wedge dv_{nn} = (\det A)^{\frac{n-1}{2}} \cdot (\det A')^{\frac{n-1}{2}} \cdot dz_{11} \wedge dz_{12} \dots \wedge dz_{nn} =$$

$$= J_{\mathbb{C}}(\Phi_3) \cdot dz_{11} \wedge dz_{12} \dots \wedge dz_{nn},$$

tenglikka ekvivalent bo'ladi, bu erda $J_{\mathbb{C}}(\Phi_3)$- Φ_3 akslantirishning kompleks yakobiani va u quyidagicha topiladi:

$$J_{\mathbb{C}}(\Phi_3) = (\det A)^{\frac{(n-1)}{2}} \cdot (\det A')^{\frac{(n-1)}{2}} = (\det A')^{(n-1)}.$$

Bulardan (1.4) shartga asosan

$$\dot{\Phi}_3 = \left|J_{\mathbb{C}}(\Phi_3)\right|^2 \dot{Z} = \left|(\det A)^{n-1}\right|^2 \dot{Z} = \frac{\dot{Z}}{\det^{n-1}(I + Z\bar{Z})}$$

tenglikka ega bo'lamiz.

Shunday qilib, yuqoridagi mulohazalarimizga ko'ra $\Phi = (h_{11},\dots,h_{nn},v_{11},\dots,v_{nn})$akslantirish uchun quyidagi

$$dh_{11} \wedge dh_{12} \wedge\dots\wedge dh_{nn} \wedge dv_{11} \wedge dv_{12} \wedge\dots\wedge dv_{nn} =$$

$$= J_{\mathbb{C}}(\Phi_2) db_{11} \wedge db_{12} \wedge\dots\wedge db_{nn} J_{\mathbb{C}}(\Phi_3) \wedge dz_{11} \wedge dz_{12} \dots \wedge dz_{nn}$$

tenglik bajarilishi va uning haqiqiy yakobiani ushbu

$$J_{\mathbb{R}}(\Phi) = \frac{1}{\det^{n+1}(I - B\bar{B}) \det^{n-1}(I + Z\bar{Z})}$$

tenglik bilan topilishi kelib chiqadi. Ma'lumki, $\mathfrak{R}_{II}(n)$va $\mathfrak{R}_{III}(n)$ sohalaring Bergman yadrolari quyidagi ko'rinishda topiladi:

$$K_{B_{II}(n)}(b,\bar{b}) = \frac{1}{V(B_{II}(n))}\frac{1}{\det^{n+1}\left(I^{(n)} - B\bar{B}\right)},$$

$$K_{B_{III}(n)}(z,\bar{z}) = \frac{1}{V(B_{III}(n))}\frac{1}{\det^{n-1}\left(I^{(n)} + Z\bar{Z}\right)},$$

bu yerda, $V(\mathfrak{R}_{II}(n))$ va $V(\mathfrak{R}_{III}(n))$ miqdorlar mos ravishda $\mathfrak{R}_{II}(n)$ va $\mathfrak{R}_{III}(n)$ sohalarning hajmlari. Demak, ushbu

$$K_B(B,Z,\bar{B},\bar{Z}) = \frac{\dfrac{1}{\det^{n+1}(I - B\bar{B})\det^{n-1}(I + Z\bar{Z})}}{V(B)} =$$

$$= \frac{1}{V(B_{II}(n))\det^{n+1}(I - B\bar{B})} ч \frac{1}{V(B_{III}(n))\det^{n-1}(I + Z\bar{Z})} =$$

$$= K_{\mathfrak{R}_{II}(n)}(B,\bar{B}) \cdot K_{\mathfrak{R}_{III}(n)}(Z,\bar{Z}).$$

munosabatlar o'rinli bo'lishi kelib chiqadi. *Teorema isbot bo'ldi.*

Bu 1-teoremadan ushbu natija kelib chiqadi.

1-natija. *Aytaylik,* $\mathfrak{R}_{II}(n)$ *va* $\mathfrak{R}_{III}(n)$ *sohalar* $B \in \mathbb{C}[n \times n]$ *va* $Z \in \mathbb{C}[n \times n]$ *o'zgaruvchili fazolardagi klassik sohalar va* $\mathfrak{R} = \mathfrak{R}_{II}(n) \times \mathfrak{R}_{III}(n)$ *bo'lsin, u holda quyidagi tenglik o'rinli bo'ladi:*

$$K_{\mathfrak{R}}(B,Z,\bar{\Psi},\bar{\Upsilon}) = K_{\mathfrak{R}_{II}(n)}(B,\bar{\Psi})K_{\mathfrak{R}_{III}(n)}(Z,\bar{\Upsilon}) \quad (1.7)$$

bu yerda, $B,\Psi \in \mathfrak{R}_{II}(n), Z,\Upsilon \in \mathfrak{R}_{III}(n)$.

$\mathfrak{R}$ sohada $d\mu$ o'lchov bo'yicha kvadrati bilan integrallanuvchi funksiyalar fazosini $L^2(\mathfrak{R})$ bilan, $H^2(\mathfrak{R})$ bilan esa $L^2(\mathfrak{R})$ sinfga golomorf davom qiluvchi uning qism fazoni belgilaymiz. 1-natijadan va Zommer-Mering teoremasiga asosan quyidagi teorema o'rinli bo'lishi kelib chiqadi.

2-teorema. *Ixtiyoriy* $f \in H^2(\mathfrak{R})$ *funksiyalar uchun*

$$f(B,Z) = \int_{\mathfrak{R}} f(\Psi,\Upsilon) K_{\mathfrak{R}}(B,Z,\overline{\Psi},\bar{\Upsilon}) d\mu, \quad (\Psi,\Upsilon) \in \mathfrak{R}$$

Bergman-Bremermann integral formulasi o'rinli.

ADABIYOTLAR RO'YXATI

1. Айзенберг Л.А, Южаков А.П. Интегралные представления и вычеты в многомерном комплексном анализе. Новосибирск: Наука, 1979. 366 с.
2. Айзенберг Л.А. Формулы Карлемана в комплексном анализе. Новосибирск: Наука, 1990. 248 с.
3. Фукс Б.А. Специалные главы теории аналитических функций многих комплексных переменных. М.: Физматгиз, 1962. – 428 с.
4. Мысливец С.Г., О ядрах Сеге и Пуассона в выпуклых областях в J^n. // Изв. вузов. Матем., 2019, № 1, 42—48
5. Хуа Ло-кен. Гармонический анализ функций многих комплексных переменных в классических областях. М.: ИЛ, 1959. – 163 с.
6. Худайберганов Г., Курбанов Б. Некоторые задачи анализа в матричных областях. // Узбекский математический журнал. 2012, № 3. с. 159-166.
7. Шабат Б. В. Введение в комплексный анализ. Ч.2. М.: Наука. 3–е изд., 1985 г. 464 с.
8. Bremermann H.J., Die Holomorphiehullen der Tuben-und Halbtubengebiete. // Math. Ann. 127, 406–423 (1954)
9. Hua L. K. and Look K. H., Theory of harmonic functions in classical domains, Sci. Sinica 8 (1959), 1031-1094.
10. Bergman S., The kernel function and conformed mapping, Amer. Math. Soc. New York, 1950.