BETA FUNKSIYA XOSSALARI VA BU FUNKSIYA YORDAMIDA TURLI MASALALARNI YECHISH

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ANNOTATSIYA

Beta funksiya kasr tartibli integral tenglamalarni va hosilalarni hisoblashda foydalaniladi. Bu funksiyalarning bir nechta xossalari kiritilgan va qisqacha ma’lumotlar berilgan.

Bugungi kunda differensial tenglamalar va matematik-fizika tenglamalari yo‘nalishlarida keng tarqalayotgan yo‘nalishlardan biri bu kasr tartibli hosilali va kasr tartibli integral tenglamalardir. Kasr tartibli tenglamalarni yechishning olimlar tomonidan bir qancha turlari o‘rganib chiqilgan. Hayotimiz davomida ko‘plab sohalar asosan fizika, kimyo, biologiya va hokazo sohalardagi jarayonlar kasr tartibli tenglamalar bilan ifodalanib, ularni o‘rganishda bizga kasr tartibli tenglamalar yordam beradi.

Kalit so‘zlar: *Beta funksiya, kasr tartibli integral, hosila, differensial tenglamalar,*

Gamma funksiya, Eyler integrali.

Biz

$$\int_0^1 x^{a-1}(1-x)^{b-1}dx \qquad (1)$$

xosmas integralni qaraylik. Integral ostidagi funksiya uchun

1) $a<1$, $b\geq 1$ bo‘lganda $x=0$ maxsus nuqta

2) $a\geq 1, b<1$ bo‘lganda $x=1$ maxsus nuqta.

3) $a<1$ $b<1$ bo‘lganda $x=1$ va $x=0$ nuqtalar maxsus nuqtalar bo‘ladi.

(1) Integral parametrga bog‘liq xosmas integraldir.(1) xosmas integralning $a>0$, $b>0$ da ya’ni

$$M=\{(a,b)\in R^2 : a\in(0,+\infty)\}$$

to‘plamda yaqinlashuvchi bo‘lishi ko‘rsatildi.

1-ta’rif: (1) integral Beta funksiyasi yoki birinchi tur Eyler integrali deb ataladi va $B(a,b)=\int_0^1 x^{a-1}(1-x)^{b-1}dx$ $(a>0,b>0)$ kabi belgilanadi.

Endi Beta funksiyaning xossalarini o‘rganaylik.

1^0 (1) $B(a,b)=\int_0^1 x^{a-1}(1-x)^{b-1}dx$ integral ixtiyoriy

$$M_0=\{(a,b)\in R^2 : a\in[a_0;+\infty), b\in[b_0;+\infty)\} \quad (a_0>0,\ b_0>0)$$

to‘plamda tekis yaqinlashuvchi bo‘ladi.

Isbot: Berilgan integralni tekis yaqinlashuvchilikka tekshirish uchun uni quyidagicha

$$\int_0^1 x^{a-1}(1-x)^{b-1}dx=\int_0^{\frac{1}{2}} x^{a-1}(1-x)^{b-1}dx+\int_{\frac{1}{2}}^1 x^{a-1}(1-x)^{b-1}dx$$

yozib olamiz.Ravshanki, $a>0$ bo‘lganda

$$\int_0^{\frac{1}{2}} x^{a-1}dx$$

integral yaqinlashuvchi, $b>0$ bo‘lganda

$$\int_{\frac{1}{2}}^{1}(1-x)^{b-1}dx$$

integral yaqinlashuvchi.Parametr a ning $a \geq a_0$ $(a_0 > 0)$ qiymatlari va $\forall b > 0$,

$\forall x \in \left(0;\frac{1}{2}\right)$ uchun $x^{a-1}(1-x)^{b-1} \leq x^{a_8-1}(1-x)^{b-1} \leq 2x^{a_0-1}$ bo‘ladi.

Veyershtrass alomatidan foydalanib

$$\int_{0}^{\frac{1}{2}} x^{a-1}(1-x)^{b-1}dx$$

Integralni tekis yaqinlashuvchi ekanligini topamiz.SHuningdek , parametr b ning $b \geq b_0$ $(b_0 > 0)$ qiymatari va $\forall a > 0$ $\forall x \in \left[\frac{1}{2},1\right)$ uchun

$$x^{a-1}(1-x)^{b-1} \leq x^{a-1}(1-x)^{b_0-1} \leq 2(1-x)^{b_0-1}$$

bo‘ladi va yana Veyrshtrass alomatiga ko‘ra

$$\int_{0}^{\frac{1}{2}} x^{a-1}(1-x)^{b-1}dx$$

Integralning tekis yaqinlashuvchiligi kelib chiqadi.Demak,

$$\int_{0}^{1} x^{a-1}(1-x)^{b-1}dx$$

Integral $a \geq a_0 > 0$ va $b \geq b_0 > 0$ bo‘lganda,ya’ni

$$M_0 = \left\{(a,b) \in R^2 : a \in [a_0;+\infty), b \in [b_0;+\infty)\right\}$$

to‘plamda tekis yaqinlashuvchi bo‘ladi.

Eslatma. $B(a,b)$ ning

$$M=\{(a,b)\in R^2 : a\in(0;+\infty), b\in(0;+\infty)\}$$

to‘plamda notekis yaqinlashuvchiligini ko‘rish qiyin emas.

2^0. $B(a,b)$ funksiya

$$M=\{(a,b)\in R^2 : a\in(0;+\infty), b\in(0;+\infty)\}$$

to‘plamda uzluksiz funksiyadir.Haqiqatan ham,

$$B(a,b)=\int_0^1 x^{a-1}(1-x)^{b-1}dx$$

integralning M_0 to‘plamda tekis yaqinlashuvchi bo‘lishidan va integral ostidagi funksiyaning $\forall(a;b)\in M$ da uzluksizligidan teoremaga asosan $B(a,b)$ funksiya

$$M=\{(a,b)\in R^2 : a\in(0;+\infty), b\in(0;+\infty)\}$$

to‘plamda uzluksiz bo‘ladi.

3^0. $\forall(a;b)\in M$ uchun $B(a,b)=B(b,a)$ bo‘ladi.Darhaqiqat

$$B(a,b)=\int_0^1 x^{a-1}(1-x)^{b-1}dx$$

integralda $x=1-t$ almashtirish bajarilsa, unda

$$B(a,b)=\int_0^1 x^{a-1}(1-x)^{b-1}dx=\int_0^1 t^{b-1}(1-t)^{a-1}dt=B(b,a)$$

bo‘lishini topamiz.

4^0. $B(a,b)$ funksiya quyidagicha ham ifodalanadi:

$$B(a,1-a)=\int_0^{+\infty}\frac{t^{a-1}}{(1+t)^{a+b}}dt=\frac{\pi}{\sin\alpha\pi} \qquad (2)$$

Haqiqatan ham, (1) integralda $x=\frac{t}{1+t}$ almashtirish bajarilsa, u holda

$$B(a,b)=\int_0^1 x^{a-1}(1-x)^{b-1}dx=\int_0^{+\infty}(\frac{t}{1+t})^{a-1}.(1-\frac{t}{1+t})^{b-1}.\frac{dt}{(1+t)^2}=\int_0^{+\infty}\frac{t^{a-1}}{(1+t)^{a+b}}dt$$

bo‘ladi.Xususan $B(\frac{1}{2};\frac{1}{2})=\pi$ bo‘lganda

$$B(a,1-a)=\int_0^{+\infty}\frac{t^{a-1}}{(1+t)^{a+b}}dt=\frac{\pi}{\sin\alpha\pi} \qquad (3)$$ bo‘ladi.

(3) munosabatdan quyidagini topamiz:

$$B(\frac{1}{2};\frac{1}{2})=\pi$$

$$B(a,b)=\frac{b-1}{a+b-1}B(a,b-1).\forall(a;b)\in M'$$

$$(M'=\{(a,b)\in R^2:a\in(0;+\infty),b\in(1;+\infty)\})$$

uchun

$$B(a,b)=\frac{b-1}{a+b-1}B(a,b-1) \qquad (4)$$

bo‘ladi.

(1) integralni bo‘laklab integrallaymiz:

$$B(a,b)=\frac{b-1}{a}[B(a,b-1)-B(a,b)]$$

$$+\frac{b-1}{a}\int_0^1 x^a(1-x)^{b-2}dx=\frac{b-1}{a}\int_0^1 x^a(1-x)^{b-2}dx$$

$(a>0,b>1)$.

Agar

$$x^a(1-x)^{b-2}=x^{a-1}[1-(1-x)](1-x)^{b-2}=x^{a-1}(1-x)^{b-2}-x^{a-1}(1-x)^{b-1}$$

ekanligini e’tiborga olsak, u holda

$$\int_0^1 x^a(1-x)^{b-2}dx = \int_0^1 x^{a-1}(1-x)^{b-2}dx - \int_0^1 x^{a-1}(1-x)^{b-1}dx \quad =$$

$B(a;b-1)-B(a,b)$ bo‘lib natijada

$$B(a,b)=\frac{b-1}{a}\left[B(a,b-1)-B(a,b)\right]$$

bo‘ladi.Bu tenglikdan esa

$$B(a,b)=\frac{b-1}{a+b-1}B(a,b-1) \qquad (a>0,b>1)$$

bo‘lishini topamiz. Xuddi shunga o‘xshash $\forall(a,b)\in M^{"}$ uchun $a=m(m\in N)$

$$B(a,b)=\frac{a-1}{a+b-1}B(a-1,b)$$

bo‘ladi.Xususan , $b=n(n\in N)$ bo‘lganda

$$B(a,b)=B(a,n)=\frac{n-1}{a+n-1}B(a,n-1)$$

bo‘lib (4) formulani takror qo‘llab, quyidagini topamiz.

$$B(a,n)=\frac{n-1}{a+n-1}.\frac{n-2}{a+n-1}-\frac{1}{n+1}B(a,1)$$

Ravshanki,

$$B(a,1)=\int_a^1 x^{a-1}dx=\frac{1}{a}$$

Demak,

$$B(a,n)=\frac{1\cdot 2\cdot ...(a-1)}{a(a+1)(a+2)...(a+n-1)} \qquad (5)$$

Agar (5) da $a=m(m\in N)$ bo‘lsa, u holda

$$B(m,n)=\frac{1\cdot 2\cdot ...(m-1)}{m(m+1)(m+2)...(m+n-1)}=\frac{(n-1)!(m-1)!}{(m+n-1)!}$$

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