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**PARABOLIK–GIPERBOLIK TIPDAGI TENGLAMALAR UCHUN
XARAKTERISTIKADAN SILJIGAN CHIZIQLARNI O‘Z
ICHIGA OLGAN QUYI YARIM SOHADA
CHEGARAVIY MASALA**

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Annotatsiya: Ushbu maqolada parabolik–giperbolik tipdagi tenglama uchun xarakteristikadan siljigan chiziqlarni o‘z ichiga olgan quyi yarim sohalarda chegaraviy masala uchun qo‘yilgan nolokal shartli chegaraviy masala yechimining mavjudligi va yagonaligi isbotlangan.

Kalit so‘zlar: Parabolik–giperbolik tipdagi tenglama, xarakteristik uchburchak, regulyar yechim, integral energiya usuli, trivial yechim, Grin funksiyasi, Volterraning ikkinchi tur integral tenglamasi, Dalamber formulasi.

**КРАЕВАЯ ЗАДАЧА В НИЖНИХ ПОЛУПОЛЯХ, ОТКЛОНЯЮЩИХСЯ
ОТ ХАРАКТЕРИСТИК ДЛЯ УРАВНЕНИЯ ПАРАБОЛО –
ГИПЕРБОЛИЧЕСКОГО ТИПА**

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Аннотация: В данной статье доказывается существование и единственность решения нелокальной условной краевой задачи для краевой задачи в нижних полуполях, содержащих линии, сдвинутые с характеристики, для уравнения параболо-гиперболического типа.

Ключевые слова: Уравнение параболо-гиперболического типа, характеристический треугольник, обычное решение, метод интегральной энергии, простое решение, функция Грина, Интегральное уравнение Вольтерра второго рода, Формула Даламбера.

**BOUNDARY VALUE PROBLEM IN LOWER SEMIFIELDS DEVIATING
FROM CHARACTERISTICS FOR A PARABOLIC–HYPERBOLIC
EQUATION**

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Annotation: This article proves the existence and uniqueness of a solution to a nonlocal conditional boundary value problem for a boundary value problem in lower semifields containing lines shifted from the characteristic for an equation of parabolic-hyperbolic type.

Key words: Equation of parabolic-hyperbolic type, characteristic triangle, usual solution, integral energy method, simple solution, Green's function, Volterra integral equation of the second kind, D'Alembert's formula.

1. Masalaning qo'yilishi

Quyidagi tenglamani qaraymiz:

$$0 = Lu \equiv \begin{cases} u_{xx} - u_y, & (x, y) \in \Omega_0, \\ u_{xx} - u_{yy}, & (x, y) \in \Omega_j \ (j = \overline{1,3}), \end{cases} \quad (1)$$

bu yerda Ω_0 soha deb $x > 0, y > 0$ bo'lganda $y = 0, x = 1, y = 1, x = 0$ to'g'ri chiziqlarda mos ravishda joylashgan AB, BB_0, B_0A_0, A_0A , kesmalar bilan chegaralangan to'rtburchak sohani, Ω_1 soha $x < 0, y > 0$ da ΔAA_0D xarakteristik uchburchakning ichida joylashgan $AK: x = \gamma_1(y)$ silliq egri chiziq va (1) tenglamaning $BP: y - x = 1$ xarakteristikasi bilan chegaralangan soha, Ω_2 soha $x > 0, y > 0$ da ΔBB_0E xarakteristik uchburchakning ichida joylashgan $AC: x = -\gamma_2(y)$ silliq egri chiziq va (1) tenglamaning $B_0M: x + y = 2$ xarakteristikasi bilan chegaralangan soha, Ω_3 soha $x < 0, y < 0$ da (1) tenglamaning $AC: y + x = 0$ va $BC: y - x = -1$ xarakteristikasi bilan chegaralangan ΔAA_0D xarakteristik uchburchak soha,

Quyidagi belgilashlarni kiritamiz: $J_1 = \{(x, y): 0 < x < 1, y = 0\}$,

$J_2 = \{(x, y): x = 0, 0 < y < 1\}$, $J_3 = \{(x, y): x = 1, 0 < y < 1\}$,

$\Omega = \Omega_0 \cup \Omega_1 \cup \Omega_2 \cup \Omega_3 \cup J_1 \cup J_2 \cup J_3, C\left(-\frac{1}{2}, \frac{1}{2}\right), D\left(-\frac{1}{2}, \frac{1}{2}\right), E\left(\frac{3}{2}, \frac{1}{2}\right)$,

$$\theta\left(\frac{x}{2}; -\frac{x}{2}\right), \left[\theta^*\left(\frac{\lambda(x)+x}{2}; \frac{\lambda(x)-x}{2}\right) \right]$$

bu yerda $\theta_2(x) \left[\theta_2^*(x) \right]$ (1) tenglamaning $(x,0) \in J_3$ nuqtadan chiquvchi xarakteristikalari bilan $AC[AN]$ xarakteristikalarining kesishish nuqtasining koordinatalari.

A-Shart. $y = -\gamma_1(x)$ va $\gamma_j(y) (j=2,3)$ - berilgan funksiyalar bo'lib quyidagi shatlarni bajarsin:

1) $\gamma_1(x)$ va $\gamma_j(y) (j=2,3)$ funksiyalar mos ravishda ΔACB va ΔAA_0D , ΔBEB_0 xarakteristik uchburchaklar ichidaqa to'liq joylashgan bo'lsin;

2) $\gamma_1(x) \in C^2(0,1)$, $\gamma_j(y) \in C^2(0,1) (j=2,3)$ tegishli bo'lsin;

3) $t \pm \gamma_j(t) (j=\overline{1,3})$ - monoton o'suvchi;

4) $\gamma_1(0) = 0$, $\gamma_2(0) = -1$, $l_1 + \gamma_1(l_1) = 1$, $l_2 - \gamma_2(l_2) = 2$, $l_3 + \gamma_3(l_3) = 1$, $l_j = const$
 $l_j \in \left(\frac{1}{2}; 1\right)$.

Ta'rif. (1) tenglamaning Ω sohadagi regulyar yechimi deb,

$$W_1 = \left\{ u : u(x, y) \in C^1(\overline{\Omega}) \cap C_{x,y}^{2,1}(\Omega_0) \cap C^2(\Omega_i), i = \overline{1,3} \right\}$$

sifga tegishli bo'lgan hamda $\Omega_i (i = \overline{0,3})$ sohada (1) tenglamani qanoatlantiruvchi $u(x, y)$ yechimga aytiladi.

I – Masala. Quyidagi shatlarni qanoatlantiruvchi (1) tenglamaning regulyar yechimi topilsin:

$$\left[u_x - u_y \right] \theta(x) + \mu(x) \left[u_x - u_y \right] \theta^*(x) = \varphi(x); \quad (2)$$

$$u|_{A_0D} = g_1(y); \quad u|_{B_0E} = g_2(y) \quad (3)$$

$$\left[u_x + u_y \right] \Big|_{AD} = p(y); \quad (4)$$

$$\left[u_x - u_y \right] \Big|_{BE} = q(y); \quad (5)$$

$$u(A) = u(B) = 0; \quad (6)$$

Bunda $\mu(x)$, $\varphi(x)$, $g(y)$, $p(y)$ va $q(y)$ - berilgan yetarlicha silliq funksiyalar.

Teorema. Agar $\mu(x) \neq -1$; $\mu(x), \varphi(x) \in C^1[0,1] (i = \overline{1,3})$, $g(y)$, $p(y)$, $q(y) \in C^2(0;1)$ va **A** shartlar bajarilsa, I - masalaning yagona regulyar yechimi mavjud bo'ladi.

Isbot. Ω_1 sohalarda Koshi masalasining yechimining ko'rinishi quyidagicha bo'ladi.

$$u(x, y) = \frac{1}{2} \left[\tau_1(x+y) + \tau_1(x-y) + \int_{x-y}^{x+y} v_1(t) dt \right], \quad 0 < x < 1$$

Bundan $\theta\left(\frac{x}{2}; -\frac{x}{2}\right)$ va $\theta^*\left(\frac{\lambda(x)+x}{2}; \frac{\lambda(x)-x}{2}\right)$ nuqtalarda quyidagi funksional

munosabatlarni olamiz:

$$(1 + \mu(x))v_1(x) = [1 + \mu(x)]\tau_1'(x) - \varphi(x), \quad 0 < x < 1 \quad (7)$$

(1) dan Ω_0 sohada $y \rightarrow +0$ limitga o'tib quyidagi tenglamani olamiz.

$$\left(\tau_1(x)\right)'' = v_1(x) \quad (8)$$

(7) ni (8) ga qo'yib $\tau_1(x)$ ga nisbatan ikkinchi tartibli oddiy differensial tenglamani hosil qilishimiz mumkin.

$$\left(\tau_1(x)\right)'' - \tau_1'(x) = -\frac{\varphi(x)}{1 + \mu(x)} \quad (9)$$

(9) tenglamani $\tau_1(0) = 0$ va $\tau_1(1) = 0$ shartlar ostida yechimning umumiy ko'rinishining shakli tasvirlanadi.

$$\tau_1(x) = \int_0^x \frac{\varphi(t)[1 - e^{x-t}]}{1 + \mu(t)} dt + \frac{e^x - 1}{e - 1} \int_0^1 \frac{\varphi(t)[e^{1-t} - 1]}{1 + \mu(t)} dt \quad (10)$$

(8) dan foydalanib, $v_1(x)$ ni topamiz

$$v_1(x) = -\int_0^x \frac{\varphi(t)e^{x-t}}{1 + \mu(t)} dt + \frac{e^x - 1}{e - 1} \int_0^1 \frac{\varphi(t)(e^{1-t} - 1)}{1 + \mu(t)} dt - \frac{\varphi(x)}{1 + \mu(x)} \quad (11)$$

Integral tenglamalar metodi bilan masalaning yechimini mavjudligini isbotlaymiz. Buning uchun biz (8)- (9) funksional munosabatlardan va Ω_0 sohada (1) tenglama uchun qo'yilgan birinchi chegaraviy masalaning yechimidan

$$u(x, y) = \int_0^1 \tau_1(t) G(x, y; t, 0) dt + \int_0^y \tau_2(t) G_t(x, y; 0, z) dz - \int_0^y \tau_3(z) G_t(0, y; 1, z) dz, \quad (12)$$

ko'rinishda bo'ladi.

Bu yerda $G(x, y; t, z) = \frac{1}{2\sqrt{\pi(y-z)}} \sum_{n=-\infty}^{\infty} \left[e^{-\frac{(x-t+2n)^2}{4(y-z)}} - e^{-\frac{(x+t+2n)^2}{4(y-z)}} \right]$ - issiqlik

o'tkazuvchanlik tenglamasi uchun birinchi chegaraviy masalaning Grin funksiyasi.

$\tau_k(y)$, $\nu_k(y)$ ($k=2,3$) funksiyalar orasidagi munosabat olish uchun bir marta x bo'yicha differensiallab:

$$u_x(x, y) = \int_0^1 \tau_1(t) G_x(x, y; t, 0) dt + \int_0^y \tau_2(z) G_{tx}(x, y; 0, z) dz - \int_0^y \tau_3(z) G_{tx}(x, y; 1, z) dz, \quad (13)$$

ni hosil qilamiz. Quyidagi $N(x, y; t, z) = \frac{1}{2\sqrt{\pi(y-z)}} \sum_{n=-\infty}^{\infty} \left[e^{-\frac{(x-t+2n)^2}{4(y-z)}} + e^{-\frac{(x+t+2n)^2}{4(y-z)}} \right]$

belgilash kiritsak,

$G_{tx}(x, y; t, z) = N_z(x, y; t, z)$, $G_x(x, y; t, z) = -N_t(x, y; t, z)$ munosabatlarga ega bo'lamiz. $u_x(0, y) = \nu_2(y)$ belgilashga ko'ra (13) dan quyidagi munosabatlarni olamiz:

$$\begin{aligned} \nu_2(y) &= \int_0^1 \tau_3'(z) N(x, y; 1, z) dz - \int_0^y \tau_1'(t) N(x, y; t, 0) dt + \int_0^y \tau_2'(z) N(0, y; 0, z) dz \\ \nu_3(y) &= \int_0^1 \tau_3'(z) N(1, y; 1, z) dz - \int_0^y \tau_1'(z) N(1, y; t, 0) dz + \int_0^y \tau_2'(z) N(1, y; 0, z) dz \end{aligned} \quad (14)$$

Ma'lumki, $u_{xx} - u_{yy} = 0$ tenglamaning umumiy yechimi

$$u(x, y) = f_1(x + y) + f_2(x - y) \quad (15)$$

ko‘rinishda bo‘ladi, bunda $f_1(\cdot), f_2(\cdot)$ - ikkinchi tartibli uzluksiz differensiallanuvchi noma‘lum funksiya.

(4) shartidan va (15) dan $f_1'(y - \gamma_2(y)) = p(y), 0 \leq y \leq l$ ga ega bo‘lamiz, tenglamadan $y - \gamma_2(y) = t$ ni yechini $y = \delta_1(t)$ ko‘rinishda izlab

$$f_1'(t) = \frac{1}{2} p(\delta_1(t)), 0 \leq y \leq l, \text{ bundan}$$

$$f_1(y) = f_1(0) + \frac{1}{2} \int_0^y p(\delta(t)) dt, 0 \leq y \leq l.$$

(3.8) shartidan va (15) dan $f_2'(y - \gamma_2(y)) = q(y), 0 \leq y \leq l$ ga ega bo‘lamiz, tenglamadan $y - \gamma_2(y) = t$ ni yechini $y = \delta_1(t)$ ko‘rinishda izlab

$$f_2'(t) = \frac{1}{2} q(\delta_1(t)), 0 \leq y \leq l, \text{ bundan}$$

$$f_2(y) = f_2(0) + \frac{1}{2} \int_0^y q(\delta(t)) dt, 0 \leq y \leq l.$$

Endi $l \leq y \leq 1$ da $u|_{A_0D} = g_1(y)$ va $u|_{B_0E} = g_2(y)$ shartni hisobga olsak,

$$\begin{cases} f_1(y) = g_1\left(\frac{y-1}{2}\right) + f_2(1), l \leq y \leq 1 \\ f_2(y) = g_2\left(\frac{2-y}{2}\right) + f_1(2), l \leq y \leq 1 \end{cases}$$

(15) ga $f_1(y)$ va $f_2(y)$ ning qiymatni qo‘yamiz va quyidagiga ega bo‘lamiz.

$$u(x, y) = \begin{cases} f_2(x - y) + \frac{1}{2} \int_0^y p(\delta(t)) dt + f_1(0), 0 \leq y \leq l, \\ f_2(x - y) + g_1\left(\frac{y-1}{2}\right) + f_2(1), l \leq y \leq 1. \\ f_1(x + y) + \frac{1}{2} \int_0^y q(\delta(t)) dt + f_2(0), 0 \leq y \leq l, \\ f_1(x + y) + g_2\left(\frac{2-y}{2}\right) - f_1(2), l \leq y \leq 1. \end{cases} \quad (16)$$

$u_y(0, y) = \tau'_i(y)$, $i = 2, 3$ ligidan, (16) tenglikni y bo'yicha bir marta differensiallab $x \rightarrow 0$ desak

$$\tau'_2(y) = \begin{cases} -f'_2(-y) + \frac{1}{2}p(\delta(y)), & 0 \leq y \leq l, \\ -f'_2(-y) + \frac{1}{2}g'_1\left(\frac{y-1}{2}\right), & l \leq y \leq 1. \end{cases} \quad (17)$$

$x \rightarrow 1$ da esa

$$\tau'_3(y) = \begin{cases} f'_1(1+y) + \frac{1}{2}q(\delta(1+y)), & 0 \leq y \leq l, \\ f'_1(1+y) - \frac{1}{2}g'_2\left(\frac{1-y}{2}\right), & l \leq y \leq 1. \end{cases} \quad (18)$$

(16) ni (17) va (18) ga qo'yib $f'_i(y)$ $i = 2, 3$ funksiya uchun $\tau_i(y)$ $i = 2, 3$ va $v_i(y)$ $i = 2, 3$ funksiyalar o'rtasida quyidagi funksional munosabatni olamiz:

$$\begin{cases} \tau'_2(y) = v_2(y) + p(\delta(y)), & 0 \leq y \leq l, x < 0 \\ \tau'_2(y) = v_2(y) + g'_1\left(\frac{y-1}{2}\right), & l \leq y \leq 1, x < 0 \end{cases} \quad (19)$$

$$\begin{cases} \tau'_3(y) = v_3(y) + q(\delta(1-y)), & 0 \leq y \leq l, x > 1 \\ \tau'_3(y) = v_3(y) + g'_2\left(\frac{1-y}{2}\right), & l \leq y \leq 1, x > 1 \end{cases} \quad (20)$$

(19) va (20) dan $v_2(y)$ va $v_3(y)$ ni (14) ga qo'ysak

$$\begin{aligned} \tau'_2(y) - \int_0^1 \tau'_3(z) N(x, y; 1, z) dz + \int_0^y \tau'_1(t) N(x, y; t, 0) dt - \int_0^y \tau'_2(z) N(0, y; 0, z) dz &= F_2(y) \\ \tau'_3(y) - \int_0^1 \tau'_3(z) N(1, y; 1, z) dz + \int_0^y \tau'_1(z) N(1, y; t, 0) dz - \int_0^y \tau'_2(z) N(1, y; 0, z) dz &= F_3(y) \end{aligned} \quad (21)$$

Bu yerda

$$\begin{aligned} F_2(y) &= \frac{1}{2}p(\delta(1-y)) - \frac{1}{2}g'_1\left(\frac{1-y}{2}\right) \\ F_3(y) &= \frac{1}{2}q(\delta(1-y)) - \frac{1}{2}g'_2\left(\frac{1-y}{2}\right) \end{aligned}$$

(21) sistemani

$$\begin{cases} \tau_2'(y) - \int_0^y \tau_2'(z) N(0, y; 0, z) dz = F_2^*(y), \\ \tau_3'(y) + \int_0^y \tau_3'(z) N(1, y; 1, z) dz = F_3^*(y). \end{cases} \quad (22)$$

ko'rinishda olamiz, bunda

$$\begin{cases} F_2^*(y) = F_2(y) + \int_0^y \tau_3'(z) N(x, y; 1, z) dz - \int_0^1 \tau_1'(t) N(x, y, t, 0) dt, \\ F_3^*(y) = F_3(y) + \int_0^y \tau_2'(z) N(1, y; 0, z) dz - \int_0^y \tau_1'(z) N(1, y, t, 0) dt \end{cases} \quad (23)$$

$$(14) \text{ sistema } |N(0, y, 0, z)| \leq \frac{2}{\sqrt{\pi|y-y_1|}} \sum_{n=1}^{\infty} \left| e^{-\frac{n^2}{|y-y_1|}} \right| \leq \text{const}$$

$|F_2^*(y)| \leq \text{const}$ bo'lgani uchun (23) sistemadan 1 – tenglamani ketma – ket yaqinlashish usuli bilan yechib

$$\tau_2(y) = F_2^*(y) + \int_0^y F_2^*(t) K(t, y) dt \quad (24)$$

ni olamiz.

$F_2^*(y)$ ni (24) ga qo'yib

$$\begin{aligned} \tau_2(y) = & F_2(y) + \int_0^y \tau_3'(z) N(x, y; 1, z) dz - \int_0^1 \tau_1'(t) N(x, y, t, 0) dt + \\ & + \int_0^y \left(F_2(y) + \int_0^t \tau_3'(z) N(x, y; 1, z) dz - \int_0^1 \tau_1'(p) N(x, y, p, 0) dp \right) K(t, y) dt \end{aligned} \quad (25)$$

Va nihoyat (25) ni (21) ning 2 – tenglamasiga qo'yamiz va $\tau_2(y)$ ga nisbatan

Volterra ikkinchi tur integral tenglamasini hosil qilamiz:

$$\begin{aligned} \tau_3'(y) - \int_0^y \tau_3'(z) N(1, y; 1, z) dz + \int_0^y \tau_1'(z) N(1, y; t, 0) dz - \int_0^z N(1, y; 0, z) dz - \int_0^y \tau_3'(z) N(x, y; 1, z) + \\ + \int_0^y \tau_3'(z) N(x, y; 1, z) dz - \int_0^1 \tau_1'(p) N(x, y, p, 0) dp K(t, y) dp + F_3(y) = F_2(y) \end{aligned} \quad (26)$$

bu yerda

$$F_3(y) = \int_0^y F_2(z)N(1, y, 0, z)dz + \int_0^y F_2(s)K(s, y)ds + \int_0^y N(1, y, 0, z) \int_0^y F_2(p)K(p, y)dp$$

(27)

(26) tenglamani yechib $\tau_2(y)$, bundan va (25) dan $\tau_1(y)$ va (26), (27) dan $\nu_1(y)$ va $\nu_2(y)$ ni topamiz. $\tau_i(y), \nu_i(y)$ ($i = \overline{1,3}$) lar ma'lum funksiyalar. Endi Ω_0 sohada I masalaning yechimini tiklashimiz mumkin, $\overline{\Omega}_i$ ($i = \overline{1,3}$) sohalarda esa Koshi masalasining yechimi bo'lgan D'alamber formulasi orqali yechim topiladi. Demak, I-masala bir qiymatli yechildi.

Teorema isbotlanadi.

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