QOVUSHQOQ-ELASTIK TO‘G‘RI TO‘RTBURCHAKLI ORTOTROP PLASTINANING ERKIN TEBRANISHI

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Annotatsiya. Maqolada o‘zgarmas qalinlikka ega bo‘lgan qovushqoq-elastik to‘g‘ri to‘rtburchakli ortotrop plastinaning erkin tebranishlari to‘g‘risidagi masalaning turli xil chegaraviy shartlar ostida matematik modeli taqdim etilgan. Kirxgof-Lyav gipotezasidan foydalangan holda o‘zgarmas qalinlikdagi qovushqoq-elastik plastinaning erkin tebranishlari to‘g‘risidagi masalaning geometrik chiziqli matematik modeli ishlab chiqilgan. Egilishning ko‘phadli approksimaksiyasiga asoslangan Bubnov-Galerkin usuli yordamida masala o‘zgarmas koeffitsientli chiziqli integral-differensial tenglamalar tizimini yechishga olib kelinadi. Zaif-singulyar yadro sifatida uchta reologik parametrli Koltunov-Rjanitsin yadrosi tanlangan. O‘zgarmas qalinlikdagi qovushqoq-elastik plastinaning erkin tebranishiga materialning reologik parametrlari, fizik-mexanik va geometrik parametrlarning ta’siri o‘rganilgan.

Kalit so‘zlar: to‘g‘ri to‘rtburchakli plastina, erkin tebranishlar, qovushqoq-elastiklik xususiyat, ortotropiya, matematik model, relaksatsiya yadrosi, integral-differensial tenglama, sonli usul.

O‘zgarmas qalinlikdagi plastinalar qurilish va texnikaning turli jabhalarida, asosan, yuk tashuvchi yupqa devorli konstruksiya elementi sifatida foydalaniladi. Ta’kidlash kerakki, ushbu ko‘rinishdagi zamonaviy yupqa devorli konstruksiyalar elementlarining mustahkamligi, uzoq muddatliligi va tuzilishiga qo‘yiladigan talablar bilan bog‘liq. An’anaviy metall materiallardan yaratilgan yupqa devorli konstruksiya elementlari bilan bir qatorda kompozit materiallardan yaratilgan konstruksiya elementlari ham foydalaniladi, bu esa materialning real xususiyatlarini inobatga olgan holda hisob ishlarini amalga oshirishni taqozo etadi. Ba’zan materiallarning qovushqoq-elastik xususiyatlarini hisobga olgan holda plastina va qobiqlarga oid masalalarni tadqiq etish o‘ta murakkab bo‘lib, bunda matematik modellashtirishdan keyin olingan integral-differensial tenglamalarni yechish masalalarini yetarli darajada qiyinchiliklar tug‘diradi. Bir tomondan, bu qaralayotgan masalada mumkin qadar haqiqiy mexanik mohiyatini aks ettirish maqsadida quriladigan matematik modellashtirishda yuzaga keladigan anchagina murakkab tenglamalarni yechish bilan bog‘liq. Boshqa tomondan esa, bu muayyan hisoblash muammolari bilan, ya’ni olinayotgan tenglamalarni yechishning munosib universal usullarining mavjud emasligi, va buning natijasi sifatida yagona hisoblash algoritmalarining mavjud bilan bog‘liqdir. Plastina va qobiqlar nazariyasining bu kabi masalalarini yechish uchun zamonaviy kompyuterlar va tayyor dasturiy mahsulotlardan keng ko‘lamda foydalanish ishlari yechimni topishda sonli usullarni yanada keng ko‘lamda qo‘llash imkonini yaratadi.

O‘zgarmas qalinlikka ega bo‘lgan plastina va qobiqlarning erkin tebranishini o‘rganishga ko‘p sondagi ilmiy ishlar bag‘ishlangan, bu ilmiy tadqiqotlar, asosan, materialning elastiklik xususiyalini hisobga olgan holda bajarilgan. Bu yo‘nalishdagi ilmiy tadqiqotlar tahlili [1, 2] berilgan.

Mavjud ilmiy tadqiqotlar tahlili shuni ko‘rsatdiki, o‘zgarmas qalinlikdagi qovushqoq-elastik yupqa devorli konstruksiyalarning plastina va qobiqlar tipidagi elementlarining turli xil chegaraviy shartlar ostidagi tadqiqotlari deyarli uchramaydi [3-5].

Mazkur ishda o‘zgarmas qalinlikdagi to‘g‘ri to‘rtburchakli ortotorop qovushqoq-elastik plastinalarning chiziqli erkin tebranishlari turli xil chegaraviy shartlarda tadqiq qilingan. [6] da taklif etilgan sonli usul mazkur masalaga moslashtirilgan va u asosida hisoblash algoritmi ishlab chiqilgan. Ishlab chiqilgan hisoblash algoritmi asosida Delphi dasturlash muhitida zamonaviya kopyuterlarda natija olish maqsadida dastur tuzilgan.

Qovushqoq-elastik ortotrop to‘g‘ri to‘rtburchakli plastinaning *a* va *b* tomonlari mos koordinata o‘qlari *Ox* va *Oy* o‘qlari bo‘yicha yo‘nalgan. Qalinligi o‘zgarmas bo‘lib, *h* ga teng. Masala geometrik chiziqli sharoitda masalaning matematik modeli an’anaviy Kirxgof-Lyav nazariyasi bo‘yicha tuzilgan, ya’ni geometrik chiziqli qovushqoq-elastik plastinaning harakat tenglamasi quyidagicha bo‘ladi [7-9]:

$$\frac{h^3}{12}\left[B_{11}\left(1-\Gamma_{11}^*\right)\frac{\partial^4 w}{\partial x^4}+\left(8B\left(1-\Gamma^*\right)+B_{12}\left(1-\Gamma_{12}^*\right)+B_{21}\left(1-\Gamma_{21}^*\right)\right)\frac{\partial^4 w}{\partial x^2\partial y^2}+\right.$$

$$\left.+B_{22}\left(1-\Gamma_{22}^*\right)\frac{\partial^4 w}{\partial y^4}\right]+\rho h\frac{\partial^2 w}{\partial t^2}=q. \tag{1}$$

Bu yerda $\Gamma^*,\Gamma_{ij}^*-\Gamma(t)$ va $\Gamma_{ij}(t)$ relaksatsiya yadrolari bilan integral operatorlar, mos ravishda

$$\Gamma^*\varphi=\int_0^t\Gamma(t-\tau)\varphi(\tau)d\tau,\ \ \Gamma_{ij}^*\varphi=\int_0^t\Gamma_{ij}(t-\tau)\varphi(\tau)d\tau,\ \ i,j=1,2,$$

$$B_{11}=\frac{E_1}{1-\mu_1\mu_2},\ B_{22}=\frac{E_2}{1-\mu_1\mu_2},\ B_{12}=B_{21}=\mu_1 B_{22}=\mu_2 B_{11},\ B=\frac{G}{2},$$

E_1, E_2 – x va y o‘qlari yo‘nalishidagi elastiklik moduli; G – siljish moduli; μ_1,μ_2 – Puasson koeffitsientlari; bu yerda va keyingi o‘rinlarda $\left(x\leftrightarrow y, 1\leftrightarrow 2\right)$ belgisi qolgan nisbatlar indekslarni davriy ravishda almashtirish tufayli yuzaga kelishini ko‘rsatadi.

Tegishli chegaraviy va boshlang‘ich shartlarga ega bo‘lgan (1) tenglama o‘zgarmas qalinlikdagi qovushqoq-elastik ortotrop to‘g‘ri to‘rtburchakli plastinaning harakat tenglamasini ifodalaydi.

$\Gamma(t),\Gamma_{ij}(t), i,j=1,2$ relaksatsiya yadrolari, quyidagi Koltunov-Rjanitsin tipidagi, zaif-singulyar yadro ko‘rinishida bo‘ladi [10]:

$$\Gamma(t)=Ae^{-\beta t}t^{\alpha-1},(0<\alpha<1),\Gamma_{ij}(\mathrm{t})=A_{ij}e^{-\beta_{ij}t}t^{\alpha_{ij}-1},\left(0<\alpha_{ij}<1\right)$$

(1) sistemaning chegaraviy shartlarini qanoatlantiruvchi yechimi quyidagi ko‘rinishda, *w* egilish bo‘yicha, izlanadi:

$$w(x,y,t)=\sum_{n=1}^{N}\sum_{m=1}^{M}w_{nm}(t)\psi_{nm}(x,y) \tag{2}$$

Bu yerda $w_{nm}=w_{nm}(t)$ - vaqtning noma’lum funksiyasi; $\psi_{nm}(x,y)$, $n=1,2,\ldots,N$; $m=1,2,\ldots,M$ - berilgan chegaraviy shartlarni qanoatlantiruvchi koordinat funksiya.

(2) ni (1) tenglamalar sistemasiga qo‘yib, quyidagi o‘lchamsiz kattaliklarni kiritib $\bar{w}=\frac{w}{h}$; $\bar{x}=\frac{x}{a}$; $\bar{y}=\frac{y}{b}$; $\bar{t}=\omega t$; $\lambda=\frac{a}{b}$; $\delta=\frac{b}{h}$; $\bar{q}=\frac{q}{E}\left(\frac{b}{h}\right)^4$; $\frac{\Gamma(t)}{\omega}$; $\frac{\Gamma_{ij}(t)}{\omega}$, $i,j=1,2$, avvalgi belgilarni saqlagan holda, Bubnov-Galerkina usulini qo‘llab, $w_{nm}=w_{nm}(t)$ noma’lum funksiyalarni topish uchun quyidagi chiziqli integral-differensial tenglamani olamiz:

$$\sum_{n=1}^{N}\sum_{m=1}^{M}c_{klnm}\ddot{w}_{nm}+\frac{1}{4\pi^4\lambda^4}\sum_{n=1}^{N}\sum_{m=1}^{M}\left[\left(1-\Gamma_{11}^*\right)f_{1klnm}+\left(1-\Gamma_{12}^*\right)f_{2klnm}+\left(1-\Gamma_{22}^*\right)f_{3klnm}+\left(1-\Gamma_{21}^*\right)f_{4klnm}+\right.$$

$$\left.+\left(1-\Gamma^*\right)f_{5klnm}\right]w_{nm}=\frac{3}{\pi^4}\left(1-\mu_1\mu_2\right)q_{kl}, \tag{3}$$

$$w_{nm}(0)=w_{0nm},\ \dot{w}_{nm}(0)=\dot{w}_{0nm},$$

Bu yerda

$$c_{klnm}=\int_0^1\int_0^1 \psi_{nm}\psi_{kl}dxdy;\ f_{1klnm}=\int_0^1\int_0^1 \Delta\psi^{IV}_{nm,xxxx}\psi_{kl}dxdy;$$

$$f_{2klnm}=\int_0^1\int_0^1 \mu_2\Delta\lambda^2\psi^{IV}_{nm,xxyy}\psi_{kl}dxdy;\ f_{3klnm}=\int_0^1\int_0^1 \frac{\lambda^4}{\Delta}\psi^{IV}_{nm,yyyy}\psi_{kl}dxdy;$$

$$f_{4klnm}=\int_0^1\int_0^1 \frac{\mu_1\lambda^2}{\Delta}\psi^{IV}_{nm,xxyy}\psi_{kl}dxdy;\ f_{5klnm}=\int_0^1\int_0^1 4(1-\mu_1\mu_2)g\lambda^2\psi^{IV}_{nm,xxyy}\psi_{kl}dxdy;$$

$$q_{kl}=q\int_0^1\int_0^1 \psi_{kl}dxdy.$$

Bu yerda $\omega=\sqrt{\dfrac{\pi^4\sqrt{E_1E_2}h^2}{3(1-\mu_1\mu_2)\rho b^4}}$ - asosiy tebranish chastotasini ifodalaydi, $\Delta=\sqrt{E_1/E_2}$.

(3) ni ikki marotaba vaqt bo‘yicha integrallaymiz:

$$\sum_{n=1}^{N}\sum_{m=1}^{M}c_{klnm}w_{nm}=\sum_{n=1}^{N}\sum_{m=1}^{M}c_{klnm}(w_{0nm}+\dot{w}_{0nm}t)-\frac{1}{4\pi^4\lambda^4}\int_0^t\int_0^\tau\left\{\sum_{n=1}^{N}\sum_{m=1}^{M}\left[\left(1-\Gamma^*_{11}\right)f_{1klnm}+\left(1-\Gamma^*_{12}\right)f_{2klnm}+\right.\right.$$
$$\left.\left.+\left(1-\Gamma^*_{22}\right)f_{3klnm}+\left(1-\Gamma^*_{21}\right)f_{4klnm}+\left(1-\Gamma^*\right)f_{5klnm}\right]w_{nm}-\frac{3}{\pi^4}(1-\mu_1\mu_2)q_{kl}\right\}d\tau ds. \qquad (4)$$

Hosil bo‘lgan ikki karrali integralni bir karrali integralga almashtiramiz va quyidagiga ega bo‘lamiz:

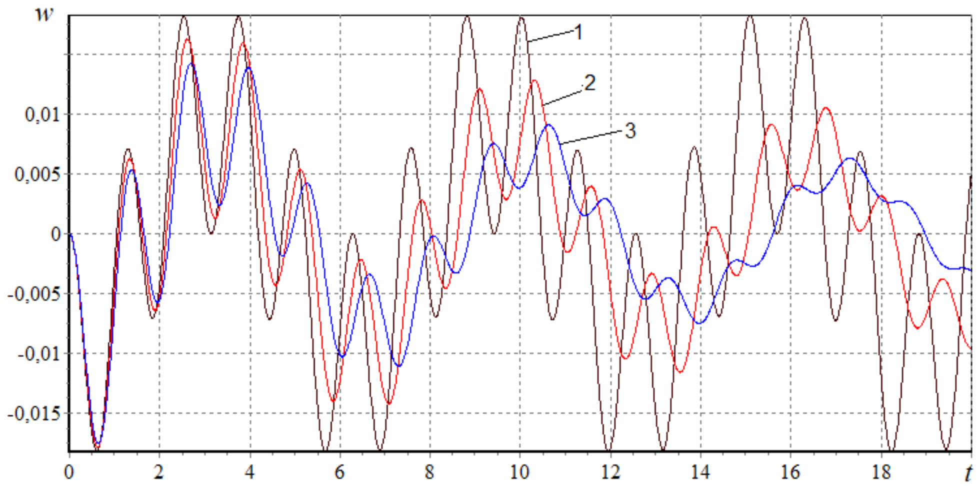
$$\sum_{n=1}^{N}\sum_{m=1}^{M}c_{klnm}w_{nm}=\sum_{n=1}^{N}\sum_{m=1}^{M}c_{klnm}(w_{0nm}+\dot{w}_{0nm}t)-\frac{1}{4\pi^4\lambda^4}\int_0^t(t-\tau)\left\{\sum_{n=1}^{N}\sum_{m=1}^{M}\left[\left(1-\Gamma^*_{11}\right)f_{1klnm}+\left(1-\Gamma^*_{12}\right)f_{2klnm}+\right.\right.$$
$$\left.\left.+\left(1-\Gamma^*_{22}\right)f_{3klnm}+\left(1-\Gamma^*_{21}\right)f_{4klnm}+\left(1-\Gamma^*\right)f_{5klnm}\right]w_{nm}-\frac{3}{\pi^4}(1-\mu_1\mu_2)q_{kl}\right\}d\tau, \qquad (5)$$

$$w_{nm}(0)=w_{0nm},\ \dot{w}_{nm}(0)=\dot{w}_{0nm},\ k=1,2,...,N;\ l=1,2,...,M.$$

Bunda $t=t_i$, $t_i=i\Delta t, i=1,2,\ldots$ (bu yerda Δt – vaqt bo‘yicha integrallash qadami) va integrallarni trapesiya ko‘rinishidagi kvadratur formulalar bilan almashtirish natijasida $w_{inm}=w_{inm}(t_i)$ funksiyalarni hisoblash uchun, umumiy holda ajralmaydigan rekurrent formulalarga ega bo‘lamiz. Ishlab chiqilgan mazkur hisoblash algoritmiga Delphi dasturlash tilida dastur tuzilgan. Dastur bo‘yicha hisob-kitob ishlari amalga oshirilgan va natijalar grafik ko‘rinishida tasvirlangan.

Olingan sistemani integrallash [6,8,9] ishdagi kvadratura formulalaridan foydalanishga asoslangan sonli usul bilan amalga oshirildi. Turli fizik-geometrik parametrlar uchun hisoblashlar natijalari 1,2 rasmlarda keltirilgan.

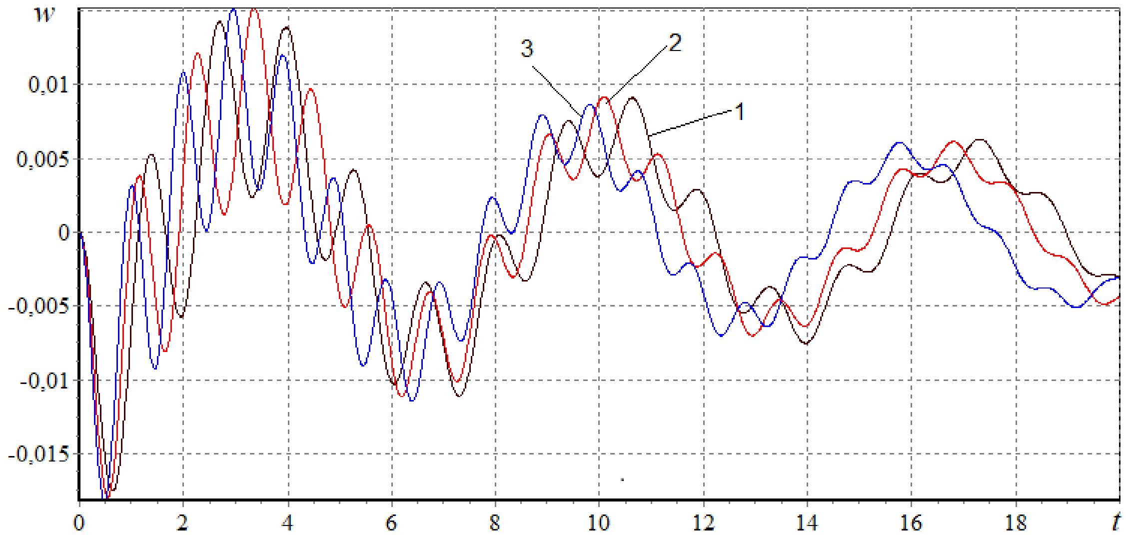
Plastina materialining qovushqoq-elastik xususiyatlarini erkin tebranish jarayoniga ta’siri 1-rasmda keltirilgan.



1-rasm. Plastina materialining qovushqoq-elastik xususiyatlarini plastina erkin tebranishiga ta'siri: A=0 (1); 0.025 (2); 0.05 (3)

Erkin tebranish plastinaning (x=0.5; y=0.5) nuqtasiga mos keladi. 1-rasmdan ko'rinib turibdiki, agar vaqtning boshlang'ich qiymatlarida natijalar deyarli ustma-ust tushgan bo'lsa, vaqt o'tgan sayin natijalar orasidagi farq oshishi kuzatiladi. Bu yana bir bor materialning qovushqoq-elastik xususiyatini hisobga olish zarurligini anglatadi.

Plastina materialining ortotropik xususiyati belgilovchi $\Delta = \sqrt{E_1 / E_2}$ parametrning erkin tebranish jarayoniga ta'siri 2-rasmda keltirilgan.



2-rasm. Plastina materialining bir jinslik bo'lmagan xususiyatlarini plastina erkin tebranishiga ta'siri: Δ=1 (1); 1.5 (2); 2 (3)

2-rasmdan ko'rinib turibdiki, anizotropiya darajasini belgilovchi Δ parametrning ortishi (1-chiziq Δ=1; 2-chiziq Δ=1,5 va 3-chiziq Δ=2) tebranishlar amplitudasining tezroq o'sishiga olib keladi.

Demak, maqolada qovushqoq-elastik ortotropik plastina erkin tebranishining matematik modeli qurilgan. Modellashtirish natjasida ajralmas (sharnirli mahkamlangan chegaraviy shartlardan tashqari) integral-differentsial tenglamaga keltirilishi ko'rsatilgan. Integral-differentsial tenglamani echishning sonli usuli taklif etilgan, bu esa qovushqoq-elastik isotrop va ortotrop plastina va qobiqlarning turli xil cheraviy shartlarda erkin tebranishini tadqiqot qilish imkonini beradi.

Foydalanilgan adabiyotlar

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