APPROXIMATION OF CHARACTERISTICS OF MULTI-PARAMETER MEASURING TRANSDUCERS

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ABSTRACT

The article deals with some issues of applying the least squares method to solving problems of polynomial approximation of the output characteristics of multi-parameter measuring systems and devices. The procedure for reducing the multidimensional problem of the best processing of experimental data of the Chebyshev approximation to a special linear programming problem is analyzed.

Keywords: *approximation, least squares method, multivariable measuring transducers, linear programming, interpolation method.*

INTRODUCTION

Multi-parameter measuring systems and devices can be divided into two classes. The first category includes those multi-parameter systems and devices for which even a single experiment to estimate a vector quantity requires a rather complex and lengthy preparation. Approximation of transformation functions for the first class of devices and systems should be carried out on the basis of the interpolation method [1,2, 3]. The second class of multi-parameter systems and devices includes those for which carrying out a significant number of experiments to estimate the vector quantity does not present any difficulties. At the same time, using multi-parameter systems and devices of the second class, it is practically possible to make a sufficiently large number of test preliminary measurements.

METHODS

Let there be an input vector quantity $\vec{x}=(x_1,...,x_i,...,x_n)\in E^n$ and an output one-dimensional quantity $u(\vec{x})=u(x_1,...,x_i,...,x_n)\in E^1$. We will approximate the unknown output function $u(\vec{x})$ by a polynomial $P_{n_1..n_i..n_n}(x_1,...,x_i,...,x_n)$, where n_i is the highest degree of the variable x_i, $(i=\overline{1,n})$.

Let the coordinate x_i of the vector quantity $\vec{x}$ and the output quantity $u(\vec{x})$ be preliminarily measured α_i times $(i=\overline{1,n})$. We will search for the approximating polynomial $P_{n_1..n_i..n_n}(x_1,...,x_i,...,x_n)$ from the condition of ensuring the minimum of the functional

$$\sum_{k_1=1}^{\alpha_1}...\sum_{k_i=1}^{\alpha_i}...\sum_{k_n=1}^{\alpha_n}\left\{\left[u\left(x_1^{(k_1)},...,x_i^{(k_i)},...,x_n^{(k_n)}\right)\right]-\left[P_{n_1..n_i..n_n}\left(x_1^{(k_1)},...,x_n^{(k_n)}\right)\right]\right\}^2,$$

where $x_i^{(k_i)}$ is k_i measurement of the variable x_i.

In this regard, we represent the polynomial $P_{n_1..n_i..n_n}(x_1,...,x_i,...,x_n)$ in the form

$$P_{n_1...n_i...n_n}(x_1,...,x_i,...,x_n)=\sum_{j_1=0}^{n_1}...\sum_{j_i=0}^{n_i}...\sum_{j_n=0}^{n_n}a_{j_1...j_i...j_n}\prod_{i=1}^{n}Q_{ji}(x_i), \qquad (1)$$

where $Q_{ji}(x_i)$ — well-defined polynomials of corresponding *n* degrees satisfying the orthogonality condition on a discrete set of points, i.e. for given measurements,

$$\sum_{k=1}^{\min(\alpha_i,\alpha_e)}Q_{ji}\left(x_i^{(k)}\right)Q_{je}\left(x_e^{(k)}\right)=0 \qquad i\neq e=\overline{1,n}.$$

From these relations, one can obtain an expression for the coefficients $a_{j_1..j_i..j_n}$

$$a_{j_1...j_i...j_n}=\sum_{k_1=1}^{\alpha_1}...\sum_{k_i=1}^{\alpha_i}...\sum_{k_n=1}^{\alpha_n}u\left(x_1^{(k_1)},...,x_n^{(k_n)}\right)\prod_{i=1}^{n}Q_{ji}\left(x_i^{(k_i)}\right)\Bigg/\prod_{i=1}^{n}\sum_{k_i=1}^{\alpha_1}Q_{ji}^2\left(x_i^{(k)}\right). \qquad (2)$$

Let us show that the determination of coefficients $a_{j_1..j_i..j_n}$ with a multi-index $(j_1,...,j_i,...,j_n)$ can be reduced to solving *n* successive problems of the one-dimensional root-mean-square approximation.

Let the output value $u(\vec{x})=u(x_1,...,x_i,...,x_n)$ be fixed in variables $x_1,x_2,...,x_{i-1},x_{i+1},...,x_n$, i.e. varies only in variable with some fixed nodes in other variables

$u\left(x_1^{(\hat{k}_1)}, x_2^{(\hat{k}_2)}, ..., x_{i-1}^{(\hat{k}_{i-1})}, x_i, x_{i+1}^{(\hat{k}_{i+1})}, ..., x_n^{(\hat{k}_n)}\right),$

where $1 \le \hat{k}_s \le \alpha_s, s = \overline{1,n}, s \ne i$.

If we introduce the designation

$$q_{\hat{j}_1 ... \hat{j}_{i-1}, j_i, \hat{j}_{i+1} ... \hat{j}_n} = \sum_{k_1=1}^{\alpha_1} u\left(x_1^{(\hat{k}_1)}, ..., x_{i-1}^{(\hat{k}_{i-1})}, x_i^{(k_i)}, x_{i+1}^{(\hat{k}_{i+1})}, ..., x_n^{(\hat{k}_n)}\right) Q_j\left(x_i^{(k_i)}\right) \Big/ \sum_{k_i=1}^{\alpha_1} Q_{ji}^2\left(x_i^{(k_i)}\right), \quad (3)$$

then $q_{\hat{j}_1 ... \hat{j}_{i-1}, j_i, \hat{j}_{i+1} ... \hat{j}_n}$ minimizes the following root-mean-square expression

$$\sum_{k_1=1}^{\alpha_1} \left[u\left(x_1^{(\hat{k}_1)}, ..., x_i^{(k_i)}, ..., x_n^{(\hat{k}_n)}\right) - \sum_{j_i=1}^{\alpha_1} q_{\hat{j}_1 .. j_i .. \hat{j}_n} Q_{ji}\left(x_i^{(k_i)}\right) \right]^2 . \quad (4)$$

It is easy to see that

$$a_{\hat{j}_1 .. j_i .. \hat{j}_n} = \sum_{k_1=1}^{\alpha_1} ... \sum_{k_{i-1}=1}^{\alpha_{i-1}} ... \sum_{k_{i+1}=1}^{\alpha_{i+1}} ... \sum_{k_n=1}^{\alpha_n} q_{\hat{j}_1 .. j_i .. \hat{j}_n} \prod_{\substack{i=1 \\ i \ne s}} Q_{js}\left(x_s^{(k_s)}\right) \Big/ \prod_{\substack{s=1 \\ s \ne i}}^{n} \sum_{k_s=1}^{\alpha_s} Q_{js}^2\left(x_s^{(k_s)}\right). \quad (5)$$

It can be seen from the last relations that if the problem of one-dimensional root-mean-square approximation is solved, then the coefficients $a_{j_1 .. j_i .. j_n}$ in the polynomial of the multidimensional root-mean-square approximation will be determined based on relation (5) [4, 5]. To calculate the errors, we calculate multidimensional polynomials according to the generalized Horner scheme

$$P_{n_1 .. n_i .. n_n}\left(x_1^{(k_1)}, ..., x_n^{(k_n)}\right) \equiv \sum_{j_1=0}^{n_1} ... \sum_{j_n=0}^{n_n} a_{j_1 ... j_n} \prod_{i=1}^{n} Q_{ji}\left(x_i^{(k_i)}\right). \quad (6)$$

When applying the multidimensional generalized Horner scheme, we take into account the fact that in the case of algebraic multidimensional polynomials

$$Q_{ji}\left(x_i^{(k_i)}\right) = \left[x_i^{(k_i)}\right]^{ji}. \quad (7)$$

Therefore

$$P_{n_1 .. n_i .. n_n}\left(x_1^{(k_1)}, ..., x_n^{(k_n)}\right) \equiv \sum_{j_1=0}^{n_1} ... \sum_{j_n=0}^{n_n} a_{j_1 ... j_n} \prod_{i=1}^{n} \left[x_i^{(k_i)}\right]^{ji} = \sum_{j_1=0}^{n_1} ... \sum_{j_n=0}^{n_n} a_{j_1 ... j_n} \prod_{i=1}^{n-1} \left[x_i^{(k_i)}\right]^{ji}, \quad (8)$$

where

$$\sum_{j_n=0}^{n_n} a_{j_n \dots j_n}\left[x_n^{(k_n)}\right] = \sum_{j_1=0}^{n_1} \dots \sum_{j_{n-1}=0}^{n_{n-1}} \prod \left[x_i^{(k_i)}\right]^{j_i} \cdot \left[_{n_n+1\,раз} \dots \left[a_{j_1 \dots j_{n-1}} \cdot n_n x_n^{(k_n)} + a_{j_1 \dots j_{n-1}} n_{n-1}\right] x_n^{(k_n)} + a_{j_1 \dots j_{n-1}} n_{n-1}\right] +$$
$$+ \dots + a_{j_1 \dots j_{n-1}} 1\Big] x_n^{(k_n)} + a_{j_1 \dots j_{n-1}} \cdot 0\Big] x_n^{(k_n)} = \left[_{n_n+1\,раз} \dots \left[x_n^{(k_n)} + \omega_{j_1 \dots j_{n-1}} \cdot n_{n-1}\right] x_n^{(k_n)} + \omega_{j_1 \dots j_{n-2}} \cdot n_{n-2}\right] x_n^{(k_n)} + \quad ,$$
$$+ \dots + \omega_{j_1 \dots j_{n-1}} \cdot 1\Big] x_n^{(k_n)} + \omega_{j_1 \dots j_{n-1}} \cdot 0\Big]$$

(9)

where $\omega_{j_1 \dots j_{n-1}} = \sum_{j_1=0}^{n_1} \dots \sum_{j_{n-1}=0}^{n_{n-1}} \prod_{i=1}^{n} \left[x_i^{(k_i)}\right]^{ji} a_{j_1 \dots j_n}$.

Thus, in the generalized multidimensional Horner scheme, one-dimensional schemes are used as many times as the dimension of the space.

In order to process information about multidimensional quantities, let us turn to the method of the best processing of experimental data (Chebyshev approximation).

The method of multidimensional interpolation ensures absolute coincidence of the signal value at the output of a multiparameter system or device with its analytical description at the nodes of a multidimensional lattice.

The least squares method minimizes the root mean square deviation error and thus makes it possible to judge the quality of processing of experimental data on average. However, when using this approximation method, significant errors are possible in individual measurements.

In this regard, specialists in the field of measurement technology involved in the processing of experimental data pay special attention to the Chebyshev approximation method [1].

RESULTS

Consider the procedure for reducing the problem of the best data processing - Chebyshev approximation to a special problem of linear programming.

Let a one-dimensional value *x* be fed to the input of the device, and there is a one-dimensional value *u* at the output.

As test signals, we will give *m* input one-dimensional signals $x_1, x_2, \dots, x_i, \dots, x_m$ and obtain the corresponding *m* output signals $u_1, u_2, \dots, u_i, \dots, u_m$.

The latter will be approximated by a generalized polynomial of the n^{th} degree

$$u(x) \approx P_n(x) = \sum_{i=1}^{n} a_i l_i(x), \qquad (10)$$

где $l_i(x), \left(i = \overline{1,n}\right)$ — arbitrary system of linearly independent functions.

In the best data processing method, the coefficients $a_i\left(i = \overline{1,n}\right)$ of the approximating polynomial should minimize the maximum deviation of the polynomial from the true value of the output signal, i.e. minimize functionality

$$\Phi(a_1, a_2 ... a_i, ... a_n) = \max_{1 \le i \le n} \left| \sum_{i=1}^{n} a_i l_i(x_k) - u_k \right|. \qquad (11)$$

Since the maximum and modulus functions are non-differentiable, it is impossible to use the necessary classical extremum condition in differential calculus to solve the experimental problem (11). To solve the extremal problem (11), we use the technique proposed by Academician L.V. Kantorovich.

Let's introduce the notation

$$z = \max_{1 \le i \le n} \left| \sum_{i=1}^{n} a_i l_i(x_k) - u_k \right|. \qquad (12)$$

We have

$$\left| \sum_{i=1}^{n} a_i l_i(x_k) - u_k \right| \le z, или \; -z \le \sum_{i=1}^{n} a_i l_i(x_k) - u_k \le z \qquad (13)$$

or

$$\sum_{i=1}^{n} a_i l_i(x_k) - z \le u_k, \sum_{i=1}^{n} a_i\left[-l_i(x_k)\right] - z \le -u_k, \quad k = \overline{1,m}. \qquad (14)$$

Since we need to minimize the maximum deviation, this means that we need to minimize *z*, subject to the fulfillment of *2m* constraints (14).

If we introduce the following vector-matrix notation

$$\vec{y} = \begin{pmatrix} a_1 \\ . \\ . \\ . \\ a_n \\ z \end{pmatrix}, \quad \vec{d} = \begin{pmatrix} 0 \\ . \\ . \\ . \\ 0 \\ 1 \end{pmatrix}, \quad k = \begin{pmatrix} l_1(x_1)...l_i(x_1)...l_n(x_1) - 1 \\ .... \\ l_1(x_m)...l_i(x_m)...l_n(x_m) - 1 \\ -l_1(x_1)...l_i(x_1)...l_n(x_1) - 1 \\ .... \\ -l_1(x_m)...l_i(x_m)...l_n(x_m) - 1 \end{pmatrix}, \quad \vec{b} = \begin{pmatrix} u_1 \\ . \\ u_m \\ -u_1 \\ . \\ -u_m \end{pmatrix}, \qquad (15)$$

then the problem of the best processing of experimental data (11) under constraints (14) can be reduced to the problem

$$\min_{\vec{y}}\left(\vec{d},\vec{y}\right) \qquad (16)$$

under restrictions $k\vec{y} \le \vec{b}$, i.e., to a problem that belongs to the linear programming environment.

DISCUSSION

The descent method is the basis of linear programming algorithms. Dual to this method, the lifting method, is used in his work on the Chebyshev approximation of multidimensional functions by J. Rice.

At the same time, the general solution of the problem of Chebyshev approximation of multidimensional functions is difficult, despite the existence of linear programming algorithms. It is difficult, first of all, because of the ambiguity of the solution of the problem of the best approximation [6]. This obstacle can be bypassed in some special cases with certain restrictions for the function being approximated (for example, the lattice of critical points of the function being approximated must be known or the function being approximated must be given at a finite number of points).

CONCLUSION

Thus, the considerable cumbersomeness of the computational scheme does not give grounds to believe that the least squares method in the multidimensional case is as effective as in the one-dimensional version. More acceptable for processing information about multidimensional quantities are, in our opinion, the interpolation method and the method of the best processing of experimental data (Chebyshev approximation).

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