CONSTRUCTION OF APPROXIMATION MODELS OF OBJECTS WITH DISTRIBUTED PARAMETERS

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ABSTRACT

The article is devoted to the development of an algorithm for the numerical implementation of transcendental and irrational transfer functions based on interpolation and variational methods. Attention is paid to the possibility of obtaining effective and simple computational equations for the most common transcendental and irrational expressions in practice. The method of modeling by the characteristics of imaginary frequencies, the use of the z-transform and the method of nonlinear approximation for modeling objects with distributed parameters are considered.

Keywords: *transcendental and irrational transfer functions, objects with distributed parameters, algorithm, interpolation and variational methods, approximation model.*

INTRODUCTION

For objects with distributed parameters described by partial differential equations and their systems, transfer functions may contain fractional powers, irrational and transcendental expressions of a complex variable, which necessitates the use of specific methods for numerically solving problems of analyzing the processes that occur in them [1, 2, 3, 4].

METHODS

Consider methods for modeling objects with distributed parameters based on the approximation of their original transfer functions.

Modeling method based on the characteristics of imaginary frequencies. Let the mathematical model be given as an irrational or transcendental transfer function $\psi(p)$. When approximating such transfer functions, one can use the transition from the complex argument p to a real variable δ, i.e. consider some function $\psi(\delta)$ (the so-called imaginary frequency characteristic). If the image $\psi(p)$ has a given value of the characteristic $\psi(\delta)$ at all points of an arbitrarily small segment of the real positive semiaxis δ (outside the singular points), then it uniquely determines the original $\varphi(t)$ at $0 \le t < \infty$.

This follows from the uniqueness of the analytic continuation of the function $\psi(\delta)$ to the entire right half-plane, including the straight line $p = k + j\omega, (k = const, -\infty < \omega < +\infty)$, and allows one to pass from approximation $\psi(\rho)$ as a function of a complex variable to approximation $\psi(\delta)$ as a function of a real variable.

Introducing the replacement $x = \dfrac{\lambda}{\delta + \lambda}(\lambda = const)$ mapping the semiaxis $0 \le \delta < \infty$ onto the segment $0 \le x \le 1$, we obtain

$$\psi(\delta) = \psi\left(\frac{\lambda}{x} - \lambda\right) = T(x). \qquad (1)$$

Using the Lagrange interpolation formula, we have

$$\begin{aligned} T(x) \approx \overline{T}(x) = T_0 + (x - x_0)\Delta_1^1 + (x - x_0)(x - x_1)\Delta_1^2 + \ldots \\ + (x - x_0)(x - x_1)\ldots(x - x_{n-1})\Delta_1^n, \end{aligned} \qquad (2)$$

where $\Delta_1^1, \ldots, \Delta_1^n$ - differences of the first order. Transforming (2), we obtain $\overline{T}(x) = h_0 + h_1 x + h_2 x^2 + \ldots + h_n (x)^n$.

Taking into account expression (1), we obtain

$$\overline{\psi}(\delta) = \sum_{i=1}^{n} h_i \left(\frac{\lambda}{\delta + \lambda}\right)^i$$

Due to the analyticity of this polynomial on the entire plane, except for the point $\delta = -\lambda$, the function $\overline{\psi}(\delta)$ can be analytically continued for complex values of the argument:

$$\overline{\psi}(p) = \sum_{i=1}^{n} h_i \left(\frac{\lambda}{p+\lambda} \right)^i .$$

The value λ in this case plays the role of the attenuation coefficient and is selected from the condition of the coincidence of the original and approximating amplitude-frequency characteristics at fixed points. To do this, at some points ω_j are calculated λ_j from the equations $a(\omega_j) = \overline{a}(\omega_j, \lambda_j), j = \overline{1, m}$, and as a calculated one, you can take the value λ from the expression

$$\lambda = \lambda_{\text{cp}} = \frac{\sum_{j=1}^{n} \lambda_j}{m}.$$

Applying a z-transform. The transfer function of a link with distributed parameters $W(x, p)$, where x is a spatial variable, can be approximated by a z-transfer function of the form

$$W(x, z) = \sum_{\ell=0}^{\ell_m} b_\ell(x) z^{-\ell}, \qquad (3)$$

which at finite ℓ_m corresponds to the difference equation

$$u(x, n) = b_0 u(0, n) + b_1 u(0, n-1) + \ldots + b_{\ell_m}(0, n-1).$$

The solution of the problem of determining the coefficients $b_\ell(x)$, as well as any approximation problem, is ambiguous and largely heuristic. The problem is solvable using frequency transfer functions when replacing $z = \exp(j\omega T), p = j\omega$, where T is the sampling period in time. This makes it possible to determine b_ℓ, based on the equations

$$W(j\omega) = \sum_{\ell=0}^{\ell_m} b_l \exp(-ej\omega T).$$

This technique is not universal, but in some cases it leads to some results [5].

Nonlinear Approximation Method. Let the impulse transient response of the object $\varphi(t)$ be given that satisfies the condition $\lim_{t\to\infty}\varphi(t)=const\neq 0$. To approximate it, we apply the approximating expression

$$\varphi(t)\approx\tilde{\varphi}(t)=a_0+e^{-\lambda t}\sum_{i=1}^{n}a_i t^{i-1}\ (i=1,2,\ldots,n), \qquad (4)$$

where λ, a_0, a_i – constant coefficients (λ – attenuation coefficient). Applying to (4) the Carson-Laplace transform, we obtain the image

$$\tilde{\psi}(p)=\sum_{i=0}^{n}h_i\frac{\lambda^i}{(p+\lambda)^i},\ \mathrm{i}=0,1,\ldots,\mathrm{n}, \qquad (5)$$

The approximating expression (5) is convenient because it is quite simply implemented by a set of algorithms that implement simple inertial elements.

Consider the procedure for determining a_i and λ. By setting the value $\lambda>0$, the coefficients a_i can be determined by one of the methods of the theory of interpolation and approximation of functions. To determine a_i, we obtain a system of equations

$$\tilde{\varphi}(t_j)=a_0+e^{-\lambda t_j}\sum_{i=1}^{n}a_i t_j^{i-1},\ (j=1,2,\ldots,n) \quad , \qquad (6)$$

(t_j – fixed argument values), whose solution gives the desired coefficients a_i. In this case, the value of the attenuation coefficient λ is usually chosen arbitrarily or from certain conditions, for example, from the conditions of fast convergence.

Consider a variational method for determining a_i, which allows choosing the order of the approximating expression, regardless of the number of points at which the function approximates. According to this method, a following function is introduced

$$\tilde{\varphi}^*(t)=\frac{\tilde{\varphi}(t)-a_0}{e^{-\lambda t}}=\sum_{i=1}^{n}a_i t^{i-1}, \qquad (7)$$

uniquely related to $\tilde{\varphi}(t)$, where

$$a_0=\lim_{t\to\infty}\varphi(t). \qquad (8)$$

a_0 is determined from (8), after which m interpolation points are selected and an approximating polynomial is constructed from the condition of the least standard deviation

$$\tilde{\varphi}^*(t) = \frac{D_1}{D_2}, \qquad (9)$$

which is the ratio of the determinants

$$D_1 = \begin{vmatrix} 0 & 1 & t & \dots & t^{n-1} \\ y_0 & c_0 & c_1 & \dots & c_{n-1} \\ y_1 & c_1 & c_2 & \dots & c_n \\ \dots & \dots & \dots & \dots & \dots \\ y_{n-1} & c_{n-1} & c_n & \dots & c_{2(n-1)} \end{vmatrix}, \; D_2 = \begin{vmatrix} c_0 & c_1 & \dots & c_{n-1} \\ c_1 & c_2 & \dots & c_n \\ \dots & \dots & \dots & \dots \\ c_{n-1} & c_n & \dots & c_{2(n-1)} \end{vmatrix}, \qquad (10)$$

where

$$c_r = \sum_{j=1}^{m} t_j^r, \; r = 0,1,2,\dots,2(n-1); \qquad \nu_\ell = \sum_{j=1}^{m} t_j^\ell \varphi^*(t_j), \; \ell = 0,1,2,\dots,(n-1), \qquad (11)$$

$\varphi^*(t) = \dfrac{\varphi(t) - a_0}{e^{-\lambda t}}, \quad j = 1,2,\dots,m$ – interpolation point numbers.

After calculating the determinants *D1* and *D2* and constructing the polynomial (9), each value a_i (i=1,2,...,n) is a coefficient at t^{i-1} and thus the function $\tilde{\varphi}^*(t)$ is determined.

Having determined the coefficients a_i and constructed the function $\tilde{\varphi}(t)$, we can estimate the error of the obtained approximation. If necessary, the error can be reduced by a new calculation with an increase in the order of the polynomial (7). An effective way to improve the resulting approximation is to refine the attenuation coefficient, which was previously chosen arbitrarily. In this case, you can use the technique, which lies in the fact that as a result of solving the equations

$$\varphi(t_j) - e^{-\lambda_j t_j} \sum_{i=1}^{n} a_i t_j^{i-1} - a_0 = 0, \; j = 1,2,\dots,m \qquad (12)$$

the value $\lambda_1, \lambda_2, ..., \lambda_m$ is determined, and the arithmetic mean value is taken as the desired value

$$\lambda = \lambda_{cp} = \frac{\sum_{j=1}^{n} \lambda_j}{m}, \qquad (13)$$

the use of which, together with the previously determined a_i, makes it possible, as a rule, to sharply reduce the approximation error. In addition, such a refinement process can be continued if the coefficients a_i are calculated again for the refined value λ or even this cycle is repeated several times. However, this path does not always lead to effective results.

RESULTS AND DISCUSSION

Consider the method of approximating the function $\varphi(t)$ using expression (6), however, we will determine λ from the condition of the minimum of the quadratic functional

$$\delta(\lambda) = \sum_{j=1}^{m} \left[\varphi(t_j) - \tilde{\varphi}(t_{j,}\lambda)\right]^2. \qquad (14)$$

As an algorithm for minimizing the functional (14) for a specific degree of the polynomial $\tilde{\varphi}(t)$, one can choose the procedure of sequential enumeration of the extinction coefficient λ with a step *h* from 0 to λ_{onm}. In expression (6), the value a_0 corresponds to the steady value of the transient response, i.e. $a_0 = \lim_{t \to \infty} \varphi(t)$ is easily determined by a given transient response.

CONCLUSION

Thus, the methods discussed above make it possible to construct approximation models of dynamic objects described by complex transfer functions of hyperbolic and irrational types, as well as high-order transfer functions.

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