PROPERTIES OF SOLUTIONS OF A NONLINEAR CROSS-DIFFUSION SYSTEM WITH VARIABLE DENSITY AND SOURCE

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Abstract. *In this work, new properties of a radially symmetric self-similar solution of the double nonlinear heat system of equations with the source were obtained. A self-similar and approximately self-similar solution was derived using the method of standard system of equations. These properties were proved by regulating the solution of the approximately self-similar system of equation relative to the parameter of the nonlinear source by adding an additional parameter. The obtained properties were verified by numerical experiments.*

Keywords: *Systems of equations, heat transfer equations, reaction-diffusion equations system, construction of a self-similar solution.*

INTRODUCTION

Consider the properties of solutions to the Cauchy problem for a system of nonlinear reaction-diffusion equations in the domain $Q=\{(t,x): t>0, x\in R^N\}$

$$\begin{cases} \dfrac{\partial u}{\partial t} = div\left(|x|^k u^{m_1-1}|\nabla u|^{p-2}\nabla u\right)+\gamma(t)v^{\beta_1} \\ \dfrac{\partial v}{\partial t} = div\left(|x|^k v^{m_2-1}|\nabla v|^{p-2}\nabla v\right)+\gamma(t)u^{\beta_2} \end{cases}, \quad (1)$$

$$\begin{cases} u(0,x)=u_0(x)\geq 0 \\ v(0,x)=v_0(x)\geq 0, x\in R^N \end{cases}, \quad (2)$$

where $k\in R, m_1, m_2>1, \beta_1, \beta_2\geq 1, p\geq 2$ are given positive numbers, $\nabla(\cdot)=grad(\cdot)$ and $u_0(x)\geq 0, v_0(x)\geq 0,\ 0<\gamma(t)\in C(0,+\infty)$. System (1) describes various physical processes in inhomogeneous two-component nonlinear media. For example, the processes of reaction-diffusion, thermal conductivity, polytropic filtration of liquids and gases with a power source equal to v^{β_1}, u^{β_2} . The cases when $k = l, p = 2, m_1 = m_2 = 0$, were considered in [1],[3]-[5].

System (1) in the region where $u = v = 0$ is degenerate, and in the region of degeneracy it may not have a classical solution. Therefore, we study weak solutions of system (1) that have a physical meaning: $0 \le u, v \in C(Q)$ and $|x|^k u^{m_1-1} |\nabla u|^{p-2} \nabla u$, and satisfy the integral identity in the sense of distribution [2]. To solve system (0.1.1), a phenomenon with a finite propagation velocity takes place. That is, there are functions $l_1(t), l_2(t)$ that satisfy $u(t,x) \equiv 0$ and $v(t,x) \equiv 0$ when $|x| \ge l_1(t), |x| \ge l_2(t)$. In case $l_1(t), l_2(t) < \infty$ for $\forall t > 0$, the solution of problem (1),(2) is called spatially localized. Surfaces $|x| = l_1(t)$ and $|x| = l_2(t)$ are called free boundary or front, respectively.

RESULTS

We proved the global solvability properties of weak solutions of system (1) using the comparison principle (see [5]). For this purpose, we construct a new system of equations using the method of standard equations [6],[8]:

$$\begin{cases} u_+(t,x) = (T+t)^{\alpha_1} \bar{f}(\xi), \\ v_+(t,x) = (T+t)^{\alpha_2} \bar{\psi}(\xi), \end{cases} \tag{3}$$

where

$$\alpha_i = -(\beta_i + 1)/(\beta_i \beta_{3-i} - 1), \xi = \phi(|x|)/[\tau(t)]^{1/p}, \tau(t) = (1/\lambda_1)(T+t)^{\lambda_1},$$

$$\lambda_i = 1 - \alpha_i(m_i + p - 3), i = 1,2.$$

In case $\alpha_1(m_1 + p - 3) = \alpha_2(m_2 + p - 3)$

$$\bar{f}(\xi) = (a - \xi^{p/(p-1)})_+^{q_2}, \bar{\psi}(\xi = (a - \xi^{p/(p-1)})_+^{q_1},$$

where

$$q_1 = \frac{p-1}{m_1 + p - 3}, q_2 = \frac{p-1}{m_2 + p - 3}, a > 0, (b)_+ = \max(0, b).$$

The Fujita type critical exponent for system (1) is the following:

$$\left(\frac{\beta_i+1}{\beta_i\beta_{3i-1}-1}\right)=\frac{N}{\left[p+\left(p+m_i-3\right)N\right]},i=1,2. \qquad (4)$$

This result contains Escobedo-Herero's result [6] for the case when $k=0,p=2,p+m_i-3=0,i=1,2$ is in (1).

Theorem 1. ***(global solvability).*** *Let's assume that* $k<p,m_i+p-3>0,$ $\beta_i>\frac{p+m_{3-i}-3}{p+m_i-3},i=1,2,-\frac{N}{p}+\frac{a_1\left(\beta_1+1\right)}{\beta_1\beta_2-1}+a_1a^{q_2\beta_1-q_1}\le 0,$ $-\frac{N}{p}+\frac{a_2\left(\beta_2+1\right)}{\beta_1\beta_2-1}+a_2a^{q_1\beta_2-q_2}\le 0,$ *and* $u_+\left(0,x\right)\ge u_0\left(x\right),v_+\left(0,x\right)\ge v_0\left(x\right),x\in R^N$. *Then for sufficiently small* $u_0\left(x\right),v_0\left(x\right)$ *we have an estimate in* Q,

$$\begin{cases}u\left(t,x\right)\le A_1u_+\left(t,x\right),\\ v\left(t,x\right)\le A_2v_+\left(t,x\right)\end{cases} \qquad (5)$$

where functions $u_+\left(t,x\right),v_+\left(t,x\right)$ are defined above and $A_1>0,A_2>0$ are constants.

Theorem 2. *Let* $k<p,m_i+p-3<0,i=1,2,\frac{\beta_1+1}{\beta_1\beta_2-1}<\frac{N}{p},\frac{\beta_2+1}{\beta_1\beta_2-1}<\frac{N}{p}$. *Then for sufficiently small* $u_0\left(x\right),v_0\left(x\right)$ *problem (1), (2) has a global solution and the following inequalities hold in* Q

$$\begin{cases}u(t,x)\le T+t)^{-N/p}\exp(-(\xi/p)^p)\\ v(t,x)\le (T+t)^{-N/p)}\exp(-(\xi/p)^p)\end{cases}\Rightarrow\left\{\xi=\phi(|x|)/(T+t)^{1/p}\right\} \qquad (5)$$

Theorem 3. *Suppose that* $q_1>0,q_2>0,\beta_1q_2>q_1,\beta_2q_1>q_2$. *Then solution* $f\left(\xi\right),g\left(\xi\right)$ *of system (0.1.9) at* $\eta\to\infty$, $\left(\eta=-\ln\left(a-\xi^{p/(p-1)}\right)\right)$ *has asymptotics*

$$\begin{cases}f(\xi)=c_1\overline{f}(\xi)(1+o(1))\\ \psi(\xi)=c_2\overline{\psi}(\xi)(1+o(1)),\end{cases}$$

where coefficients c_1,c_2 satisfy a system of algebraic equations

$$\begin{cases} -(q_1)^{p-1} c_1^{\,m_1+p-3} + a_1 \gamma^{-p} = 0, \\ -(q_2)^{p-1} c_2^{\,m_2+p-3} + a_2 \gamma^{-p} = 0. \end{cases}$$

A graphical representation of a special algorithm built for the parameter values of all the results obtained looked as follows:

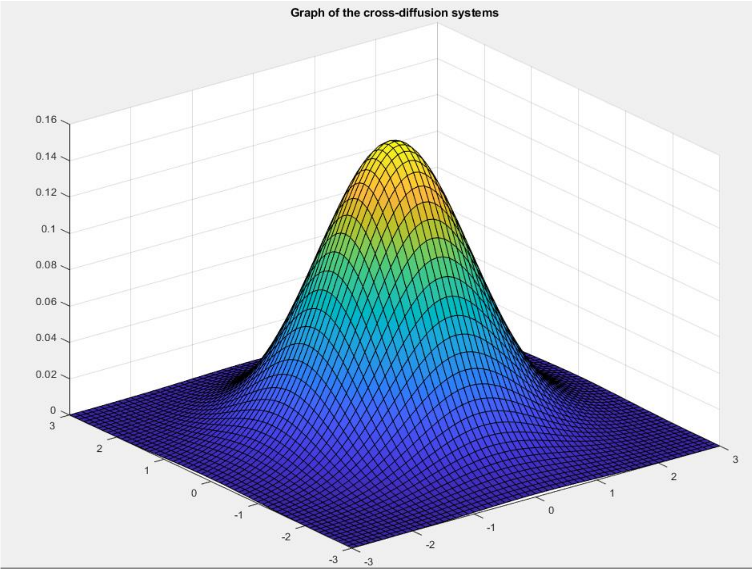

Figure 1. Graph of the cross-diffusion systems, parameters *p=2.5, m1=1.4, m2=1.5*

CONCLUSION

The results of the search for a solution to the given problem (1)-(2) show that in order to study the processes of nonlinear heat transfer, nonlinear heat conduction, it is very important to analyze self-similar or approximately self-similar solutions and find qualitative properties of systems of equations, find compact asymptotically supported states of non-destructive solutions of systems of equations. Self-similar model and approximately self-similar when searching for solutions to self-similar models, new effects related to the problem were observed. It is known from computational experiments that a nonlinear decomposition method and a self-similar method are constructed in a practical way, when searching for numerical solutions of nonlinear standard systems of equations, good results of generating iterative methods by Newton and Picard methods lead to numerically computable results of their solutions. Depending on the values of the parameters found, one can see the influence of the heat transfer rate, and cases of spatial localization.

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