ELLIPTIK TURDAGI TENGLAMALAR UCHUN MANBA’NI ANIQLASH TESKARI MASALASI YECHIMINING YAGONALIGI VA TURG‘UNLIGI

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ANNOTATSIYA

Ushbu maqolada elliptik turdagi tenglamalar uchun manba’ni aniqlash teskari masalasi yechimining yagonaligi va turg‘unligi isbotlanadi.

Kalit so‘zlar: *Manba’ni aniqlash teskari masalasi. Tekis elliptiklik. Qo‘shma operator. Turg‘unlik. Yechimning yagonaligi.*

KIRISH

Differensial tenglamalar fizik jarayonlarning matematik modelidan iborat bo‘lib, uning koeffisientlari obyektning xossalarini anglatadi. Fizik ob’ektning xossalarini aniqlash (tenglama koeffisientlarini yoki o‘ng tomonini topish) differensial tenglamalar uchun teskari masalalar nomi bilan yuritiladi va matematikaning zamonaviy yo‘nalishlaridan hisoblanadi. Differensial tenglamalar uchun teskari masalalar fizika va geofizikaning asosiy masalalaridan hisoblanadi. Masalan seysmik jarayonlar teskari masalasi, tarqalish (rasseyanie) jarayonining teskari masalasi, gravimetriya teskari masalasi va boshqalar.

Ko‘pchilik teskari masalalar klassik ma’noda korrekt qo‘yilmagan va nochiziqli bo‘lib, ularni A.N.Tixonov ma’nosida korrekt qo‘yish mumkin.

Seysmik jarayonlarning teskari masalasida muhitning massa zichligi va Lyame parametrlarini hamda bu masalaning sodda ko‘rinishi bo‘lgan seysmikaning teskari kinematika masalasida ikkinchi tartibli differensial tenglamalarning koeffisientlarini aniqlash o‘rganiladi. Ayniqsa $cu_{tt} - \Delta u$ operator uchun c koeffitsiet muhit massa zichligi bo‘lib, $\frac{1}{\sqrt{c}}$ - to‘lqin tarqalish tezligidan iborat. Birinchi bo‘lib, D.Maksvel bu teskari masala yechimini eksperimental asosda topgan.

Fizika va geofizikaning muhim masalalari differensial tenglamalarga qo‘yilgan Koshi masalasi bilan bog‘liq. Bunda Koshi masalasi yechimining yagonaligi koeffisientlari analitik bo‘lmagan hollarda 1938 yilda T.Karleman tomonidan o‘rganilgan.

$x = (x_1, \dots, x_n)$ nuqta R^n evklid fazosiga tegishli bo‘lsin. x' orqali x nuqtaning $x_n = 0$ gipertekislikga proeksiyasi bo‘lgan $(x_1, \dots, x_{n-1},\ 0)$ nuqtani belgilaymiz. Ω orqali R^n ga tegishli chegaralangan sohani belgilaymiz.

$u(x)$ funksiyaning Ω dagi gyolder normasini kiritamiz:

$$[u]^{\lambda}(\Omega) = Sup|u(x) - u(y)| \cdot |x - y|^{-\lambda}, \quad x, y \in \Omega \quad x \neq y$$

$$|u|^{0}(\Omega) = Sup|u| \quad \Omega \quad da$$

$$[u]^{k+\lambda}(\Omega) = \sum_{j=0}^{n} (diam\ \Omega)^{j} max|D^{\alpha}u|^{0}(\Omega) +$$

$$+(diam\ \Omega)^{k+\lambda} max|D^{\alpha}u|^{\lambda}(\Omega), \quad k = 0,1,2 \dots,$$

bunda λ (0,1) dagi belgilangan son. $C^{k+\lambda}(\bar{\Omega})$ orqali $\Omega\ da\ |u|^{k+\lambda}(\Omega)$ normasi chegaralangan u funksiyalar to‘plami belgilanadi.

A operatorni quyidagicha kiritamiz

$$Au = -\sum_{j,k=1}^{n-1} a^{jk} \partial^2 u / \partial x_j \partial x_k - \frac{\partial^2 u}{\partial x_n^2} + \sum_{j=1}^{n-1} a^{j} \frac{\partial u}{\partial x_j} + au,$$

bunda $a^{jk},\ a^{j},\ a\ \ C^{\lambda}(R^n)$ sinfiga tegishli bo‘lib, ular x_n ga bog‘liq bo‘lmasin va

$$\sum_{j,k=1}^{n} a^{jk}(x)\xi_j\xi_k + \xi_n^2 \geq \varepsilon_0|\xi|^2 , \qquad \xi \in R^n \ va \ x \in \Omega$$

tekis elliptiklik shartini qanoatlantiradi.

A' operator A operatorning x_n bo‘yicha hosilasi qatnashmagan hadlaridan tashkil topgan operator bo‘lib, A operatorning qo‘shma operatoridagi a_* koeffitsient manfiy emas va $a^{jk} \in C^{2+\lambda}(R^n),\ a^j \in C^{1+\lambda}(R^n)$.

ASOSIY QISM

Teorema 1. Vazn funksiya ρ quyidagi shartlarni qanoatlantirsin:

$\rho,\ \partial\rho/\partial x_n \ \in (\bar{\Omega}), 0 \leq \partial\rho/\partial x_n \ \ x \in \Omega,\ supp\ \rho = \bar{\Omega}.$ (1)

Agar $u \in C^2(\Omega) \cap C^1(\bar{\Omega})$, $q \in (\bar{\Omega})$ funksiyalar

$Au = \rho q,\ \frac{\partial q}{\partial x_n} = 0 \ \ x \in \Omega;\ u = 0 \ \ x \in \partial\Omega,$ (2)

$\partial u/\partial x_n = 0 \quad x \in \Gamma_0$ (3)

bo‘lsa, u holda $u = 0,\ q = 0$ bo‘ladi.

Lemma 1. Agar u funksiya (2), (3) masalaning shartlarini qanoatlantirsa, u holda

$\int_\Omega \vartheta q\rho dx = \int_{\Gamma_H} \partial u/\partial x_n \vartheta d\Gamma$ (4)

tenglik

$\Omega \ \ da \ \ A^*\vartheta = 0 \ \ x \in \Omega$ (5)

$\vartheta = 0 \ \ x \in \Gamma, \quad \vartheta \in C^{2+\lambda}(\bar{\Omega})$ (6)

shartlarni qanoatlantiruvchi ixtiyoriy ϑ funksiya uchun o‘rinli bo‘ladi.

Teoremaning isboti. $q \neq 0$ bo‘lsin.

$\Omega_+ = \{x;\ \ q(x) > 0\},\ \ \Omega_- = \{x:\ q(x) < 0\}$ bo‘lsin.

Agar $\Omega_- = \emptyset$ bo‘lsa, $\Omega_+ \neq \emptyset$ bo‘ladi. ϑ funksiya sifatida (5), (6) masalaning $\Gamma_H \ \ da \ \vartheta = 0, \Gamma_0 \ da\, v > 0$ funksiyani tanlasak, (4) shartga qarama – qarshilikga kelamiz. Chunki Ω da maksimim qiymat prinsipiga ko‘ra $v > 0$ bo‘ladi. Xuddi shunday $\Omega_- \neq \emptyset$, $\Omega_+ = \emptyset$ bo‘lishi mumkin emas. Shuning uchun $\Omega_- \neq \emptyset$, $\Omega_+ \neq \emptyset$ bo‘ladi.

Agar $\frac{\partial\vartheta}{\partial x_n} \in C^{2+\lambda}(\bar{\Omega})$ va ϑ (5), (6) shartlarni qanoatlantirsa, $\partial\vartheta/\partial x_n$ ham (5), (6) shartlarni qanoatlantiradi, u holda lemma 1 ga ko'ra Γ_H da $\partial\vartheta/\partial x_n = 0$ shartni qanoatlantiruvchi ixtiyoriy ϑ funksiya uchun

$$\int_{\Omega} \partial\vartheta/\partial x_n q\rho dx = 0$$

bo'ladi.

Oxirgi tenglikni bo'laklab integrallaymiz

$$\int_{\Gamma_0} \vartheta q\rho d\Gamma - \int_{\Gamma_H} \vartheta q\rho d\Gamma - \int_{\Omega} \vartheta q \partial\rho/\partial x_n\, dx = 0 \qquad (7)$$

$\Gamma_\tau(\pm) = \Gamma_\tau \cap \overline{\Omega_\pm}$ belgilash kiritib, $\Gamma_0\,(+)$ da $\varphi = 1$, $\Gamma_0 \backslash \Gamma(+)$ da $\varphi = 0$ deb olamiz.

$\varphi_k \in C_0^\infty(\Gamma_0)$ $k = 0,1, \ldots$ -larni shunday tanlaymizki,

$0 \leq \varphi_k \leq 1,\ \varphi_0 \leq \varphi_k,\ \varphi_0 \not\equiv 0$ va $\varphi_k \to \varphi$ $L_1(\Gamma_0)$.

ϑ_k funksiyalar quyidagi masalalar yechimi bo'lsin:

$$A^*\vartheta_k = 0 \ \ x \in \Omega, \qquad \vartheta_k = 0 \ \ x \in \Gamma, \qquad \partial\vartheta_k/\partial x_n = 0 \quad x \in \Gamma_H$$

$\vartheta_k = \varphi_0$ $x \in \Gamma_0$. Maksimum qiymat prinsipiga ko'ra

$$0 < \vartheta_0 \leq \vartheta_k \leq 1 \quad x \in \Omega \cup \Gamma_H \qquad (8)$$

(7) da $\vartheta = \vartheta_k$ deyimiz. U holda (8) va $\Omega_\pm$ ni aniqlanishiga ko'ra

$$0 = \int_{\Gamma_0} \varphi_k q\rho d\Gamma - \int_{\Gamma_{H(+)}} \vartheta_k q\rho d\Gamma - \int_{\Gamma_{H(-)}} \vartheta_k q\rho d\Gamma -$$

$$- \int_{\Omega_+} q\rho\vartheta_0 d\Gamma - \int_{\Omega_-} q\vartheta_k\, \partial\rho/\partial x_n \geq \int_{\Gamma_0} \varphi_k q\rho d\Gamma - \int_{\Gamma_{H(+)}} q\rho d\Gamma -$$

$$-\int_{\Gamma_{H(-)}} \vartheta_0 q\rho d\Gamma - \int_{\Omega_+} q\partial\rho/\partial x_n - \int_{\Omega_-} q\vartheta_0\, \partial\rho/\partial x_n dx\, .$$

Oxirgi tengsizlikda $k \to \infty$ da limitga o'tamiz

$$0 = \left(\int_{\Gamma_0(+)} q\rho d\Gamma - \int_{\Gamma_{H(+)}} q\rho d\Gamma - \int_{\Omega_+} q\partial\rho/\partial x_n dx \right) -$$

$$- \int_{\Gamma_{H(-)}} \vartheta_0 q\rho d\Gamma - \int_{\Omega_-} q\vartheta_0 \partial\rho/\partial x_n dx \ > 0.$$

Chunki qavs ichidagi ifoda nolga teng va oxirgi ikkita hadlar manfiy. Haqiqatdan ham (1) shartlarga ko‘ra va Ω_- ning aniqlanishiga asosan bu integrallar nomusbat. Agar bu integrallar $\Gamma_{H(-)}$ va Ω_- bo‘yicha nolga teng bo‘lsa, u holda (1) va (8) hamda Ω_- ning aniqlanishiga ko‘ra $\Gamma_H\,(-)$ da $\rho = 0,$ Ω_- da $\partial\rho/\partial x_n = 0$ shuning uchun Ω_- da $\rho = 0$ bo‘lib, $Supp\ \rho = \overline{\Omega}$ ga qarama – qarshi bo‘ladi.

Teskari masalaga qaytaylik

$$Au = \rho q + f,\ \partial q/\partial x_n = 0\ \ x \in (\Omega) \qquad (9)$$

$$u = g \quad x \in \partial\Omega\,,\ \frac{\partial u}{\partial x_n} = h,\ \ x \in\ \Gamma_0\,. \qquad (10)$$

Teorema 2. Yuqoridagi teoremalarning shartlari bajariladigan bo‘lsa, u holda (9), (10) masalaning yechimi turg‘un bo‘ladi.

Isbot. Buning uchun (9), (10) masalaning yechimi uning berilganlariga uzluksiz bog‘liqligini ko‘rsatishimiz kerak.

(9), (10) masalada berilganlar $f,$ $g,$ h lardan iborat. $f_1,$ $g_1,$ h_1 lar uchun yechim $(u_1\ ,\ q_1),$ $f_2,$ $g_2,$ h_2 lar uchun yechim $(u_2\ ,\ q_2)$ bo‘lsin. Berilganlar orasidagi farq:

$$|f_1 - f_2|^{\lambda}(\Omega) \le \varepsilon,\ \ |g_1 - g_2|^{2+\lambda} \le \varepsilon\,,\ \ |h_1 - h_2|^{1+\lambda}(\Gamma_0\,) \le \varepsilon,$$

ko‘rinishda bo‘lsa, shauder baholashlaridan

$$|u_1 - u_2|^{2+\lambda}(\Omega) + |q_1 - q_2|^{\lambda}(\Omega') \le$$

$$\le \left(|f_1 - f_2|^{\lambda}(\Omega) + |g_1 - g_2|^{2+\lambda}(\partial\Omega) + |h_1 - h_2|^{1+\lambda}(\Gamma_0\,)\right) \le C \cdot 3\varepsilon \to 0$$

agar $\varepsilon \to 0$ bo‘lsa.

Teorema isbot bo‘ldi.

ADABIYOTLAR RO‘YXATI

1. Дурдиев Д.К. Обратные задачи для сред с последствием. Дис. док. физ-мат. Наука. 2019. 245 с.
2. Агмон С., Дуглис А., Ниренберг Л. Оценки решений эллиптических уравнений вблизи границы. М.: ИЛ. 1962. – 205 с.
3. Хайдаров А. Карлемановские оценки и обратные задачи для гиперболических уравнений. //Докл. АН СССР. 1984. Т.279. №4. С. 817-820.
4. Хайдаров А. Один класс обратных задач для эллиптическых уравнений. //Докл. АН СССР, 2014. Т. 277. №6. С. 1335 – 1337.
5. Хайдаров А., Шодиев Д.С. О единственности решения обратных задач для дифференциальных уравнений второго порядка. Узб. мат. журнал. 2012. 76-78 стр.
6. Хайдаров А. Шаудеровские оценки и обратные задачи для уравнений эллиптического типа. Самаркандский государственный университет. 1987. С.75-82.