IKKI KARRALI INTEGRALNI MAPLE DASTURI YORDAMIDA HISOBLASH USULLARI

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ANNOTATSIYA

Karrali integrallarni MAPLE matematik paketi yordamida yechish. MAPLE matematik paketi yordamida karrali integrallarni yechishning imkoniyatlari va qulayliklari. Karrali integrallarni MAPLE matematik yordamida yechishni ko‘rgazmalilikligi va karrali integrallarni MAPLE matematik paketida yechishning odatdagi usullardan farqlari.

Kalit so‘zlar: *MAPLE, ikki karrali integrallar, yuza, hajim, soha, grafik.*

ABSTRACT

Solving multiple integrals using the MAPLE mathematical package. Opportunities and convenience of solving multiple integrals using the MAPLE mathematical package. Visualization of solving multiple integrals using MAPLE math and differences from usual methods of solving multiple integrals in MAPLE math package.

Key words: *MAPLE, double integrals, surface, volume, area, graph*

KIRISH

Matematik analiz fanining muhim bo‘limlaridan bittasi karrali integrallar hisoblanadi. Karrali integral tekislikning ma’lum sohasida uch o‘lchovli yoki n o‘lchovli fazoda berilgan funksiyalardan olingan integral. Bu yerda talabalarda shunday muamo yuzaga keladiki, qandaydir D sohada aniqlangan ikki o‘zgaruvchili funksiyaning grafigini tassavur qilish va hosil bo‘lgan jismni doskada ifodalash bir muncha qiyinchiliklar tug‘diradi. Bunday hollarda ayniqsa uch o‘lchovli fazoda jismni tasvirlash uchun matematik paketlardan foydalanish samarali hisoblanadi. Hozirgi kunda eng zamonaviy paketlardan biri MAPLE shu muamolarni hal qilishda muqobil yechim hisoblanadi. Dastlab biz ikki karrali integralga ta’rif beramiz va ikki karrali integralni MAPLE matematik paketi yordamida yechishning bir nechta usullarini keltirib, ularga doir bir nechta misollar yechimlarini ko‘rsatamiz.

Ta’rif: Oxy tekislikning yopiq D sohasida $z = f(x, y)$ funksiya aniqlangan va uzluksiz bo‘lsin. D sohani ixtiyoriy ravishda umumiy ichki nuqtalarga ega bo‘lmagan va yuzalari ΔS_i ga teng bo‘lgan n ta D_i $(i = \overline{1, n})$ elementar sohalarga bo‘lamiz. Har bir D_i sohada ixtiyoriy $P(x_i; y_i)$ nuqtani tanlaymiz, $z = f(x, y)$ funksiyaning bu nuqtadagi qiymati $f(x_i, y_i)$ ni hisoblab, uni ΔS_i ga ko‘paytiramiz va barcha bunday ko‘paytmalarning yig‘indisini tuzamiz:

$$I_n = \sum_{i=1}^{n} f(x_i, y_i)\Delta S_i$$

Bu yig‘indiga $f(x, y)$ funksiyaning D sohadagi integral yig‘indisi deyiladi.

D_i soha chegaraviy nuqtalari orasidagi masofalarning eng kattasiga shu yuzaning diametri deyiladi va d_i bilan belgilanadi, bunda $n \to \infty$ da $d_i \to 0$. Agar (1.1) integral yig‘indining $\max d_i \to 0$ dagi chekli limiti D sohani bo‘laklarga bo‘lish usuliga va bu bo‘laklarda $P(x_i; y_i)$ nuqtani tanlash usuliga bog‘liq bo‘lmagan holda mavjud bo‘lsa, bu limitga $f(x,y)$ funksiyadan D soha bo‘yicha olingan ikki karrali integral deyiladi va $\iint_D f(x,y)\,dS$ bilan belgilanadi:

$$\iint_D f(x,y)\,dS = \lim_{\max d_i \to 0} \sum_{i=1}^{n} f(x_i, y_i)\Delta S,$$

yoki,

$$\iint_D f(x,y)\,dx\,dy = \lim_{\max d_i \to 0} \sum_{i=1}^{n} f(x_i, y_i)\Delta x_i \cdot y_i$$

bo‘ladi.

1. Ikki karrali integralni int() buyrug‘i yordamida hisoblash.

Bu buyruq quydagi tartibda yozilsa bizga ikki karrali integralni hisoblab beradi.

Int funksiyasi ichiga funksiya va uning o‘zgarish oraliqlarini kiritamiz va uni yana shunga tenglab natijani olamiz faqat bunda integralni hisoblash jarayoni ko‘rinmaydi. Buni ko‘rish uchun “simplify” buyrug‘idan foydalanamiz va natijalarni olamiz.

2. Ikki karrali integralni Student paketidagi Doubleint() funksiyasi yordamida hisoblash.

Doubleint( ) funksiyasi yordamida ikki karrali integralni hisoblash uchun with(student): Doubleint buyrugʻi ichiga funksiya va oʻzgaruvchilarning oʻzgarish oraligʻi kiritib natija olamiz.

3. Ikki karrali integralni Student[MultivariateCalculus] paketining MultiInt() funksiyasi yordamida hisoblash.

Student[MultivariateCalculus] paketidagi MultiInt( ) funksiyasi yordamida ikki karrali integral quydagicha hisoblanadi. MultiInt( ) funksiyasi ichiga ikki karrali integral ostidagi funksiya, o‘zgaruvchilarning o‘zgarish oralig‘i va oxirida outpat=steps deb natijani olamiz.

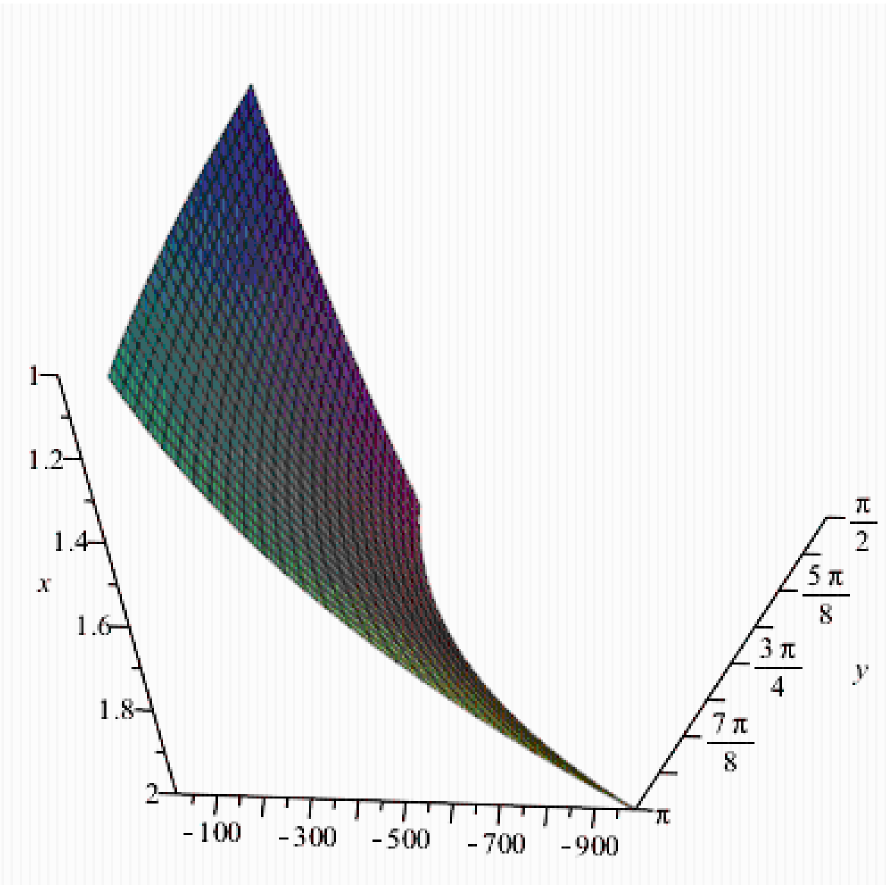
4. Ikki karrali integralni VectorCalculus paketining int() funksiyasi yordamida hisoblash.

VectorCalculus paketining int() funksiyasi bilan hisoblashda with(VectorCalculus) va int buyrug'idan foydalanamiz. Int() funksiyasi ichiga ikki karrali integral ostidagi funksiyani va Region funksiya ichiga o'zgarish oraliqlarini kiritib natijani olamiz.

Endi shularga oid misollarni ko'rib chiqamiz.

1-misol: $\iint\limits_D \left(xy-4x^3y^3\right)dxdy,\ D: x=1,\ y=x^3,\ y=-\sqrt{x}$ hisoblang.

$> plot3d\left(x\cdot y-4\cdot x^3\cdot y^3, x=1..2, y=\frac{\pi}{2}..\pi, axes=frame\right)$



Ikki karrali integralni int() buyrug'i yordamida hisoblash.

$>$

$$Int\left(Int\left('(x\cdot y-4x^3\cdot y^3)', y=-x^{\frac{1}{2}}..x^2\right), x=0..1\right)=int\left(int\left(x\cdot y-4x^3\cdot y^3, y=-x^{\frac{1}{2}}..x^2\right), x=0..1\right);$$

$$\int_0^1 \int_{-\sqrt{x}}^{x^2} (-4x^3 y^3 + xy)\, dy\, dx = 0$$

$$Int\left('(x \cdot y - 4x^3 \cdot y^3)', y = -x^{\frac{1}{2}} .. x^2\right) = int\left(x \cdot y - 4x^3 \cdot y^3, y = -x^{\frac{1}{2}} .. x^2\right);$$

$$\int_{-\sqrt{x}}^{x^2} (-4x^3 y^3 + xy)\, dy = -x^3 (x^8 - x^2) + \frac{1}{2} x (x^4 - x)$$

$$simplify(rhs(\%));$$

$$-x^{11} + \frac{3}{2} x^5 - \frac{1}{2} x^2$$

$$Int(\%, x = 0..1) = int(\%, x = 0..1);$$

$$\int_0^1 \left(-x^{11} + \frac{3}{2} x^5 - \frac{1}{2} x^2\right) dx = 0$$

Ikki karrali integralni Student paketidagi Doubleint() funksiyasi yordamida hisoblash.

$$> with(student) : Doubleint\left((x \cdot y - 4x^3 \cdot y^3), y = -x^{\frac{1}{2}} .. x^2, x = 0..1\right) : \% = value(\%);$$

$$\int_0^1 \int_{-\sqrt{x}}^{x^2} (-4x^3 y^3 + xy)\, dy\, dx = 0$$

Ikki karrali integralni Student[MultivariateCalculus] paketining MultiInt() funksiyasi yordamida hisoblash.

$$with(Student[MultivariateCalculus]);$$

$$MultiInt\left(x \cdot y - 4x^3 \cdot y^3, y = -x^{\frac{1}{2}} .. x^2, x = 0..1, output = steps\right);$$

[`.`, ApproximateInt, ApproximateIntTutor, AreParallel, AreSkew, CenterOfMass, ChangeOfVariables, Contains, CrossSection, CrossSectionTutor, Del, DirectionalDerivative, DirectionalDerivativeTutor, Distance, Equal, FunctionAverage, GetDimension, GetDirection, GetIntersection, GetNormal, GetPlot, GetPoint, GetRepresentation, Gradient, GradientTutor, Intersects, Jacobian, LagrangeMultipliers, Line, MultiInt, Nabla, Plane, Revert, SecondDerivativeTest, SurfaceArea, TaylorApproximation, TaylorApproximationTutor]

$$\int_0^1 \int_{-\sqrt{x}}^{x^2} \left(-4x^3y^3 + xy\right) dy\, dx$$

$$= \int_0^1 \left(\left(-y^4x^3 + \frac{1}{2}y^2x \right) \Big|_{y=-\sqrt{x}\,..x^2} \right) dx$$

$$= \int_0^1 \left(-x^3\left(x^8 - x^2\right) + \frac{x\left(x^4 - x\right)}{2} \right) dx$$

$$= \left(-\frac{1}{12}x^{12} + \frac{1}{4}x^6 - \frac{1}{6}x^3 \right) \Big|_{x=0\,..1}$$

Ikki karrali integralni VectorCalculus paketining int() funksiyasi yordamida hisoblash.

$$with(VectorCalculus):\ int\left(x\cdot y - 4x^3\cdot y^3, [x,y] = Region\left(0\,..\,1, -x^{\frac{1}{2}}\,..x^2\right)\right);$$

Demak, yuqoridagi 4 ta usulda bizga berilgan

$$\iint_D \left(xy - 4x^3y^3\right) dxdy,\ \ D: x = 1,\ y = x^3,\ y = -\sqrt{x}$$

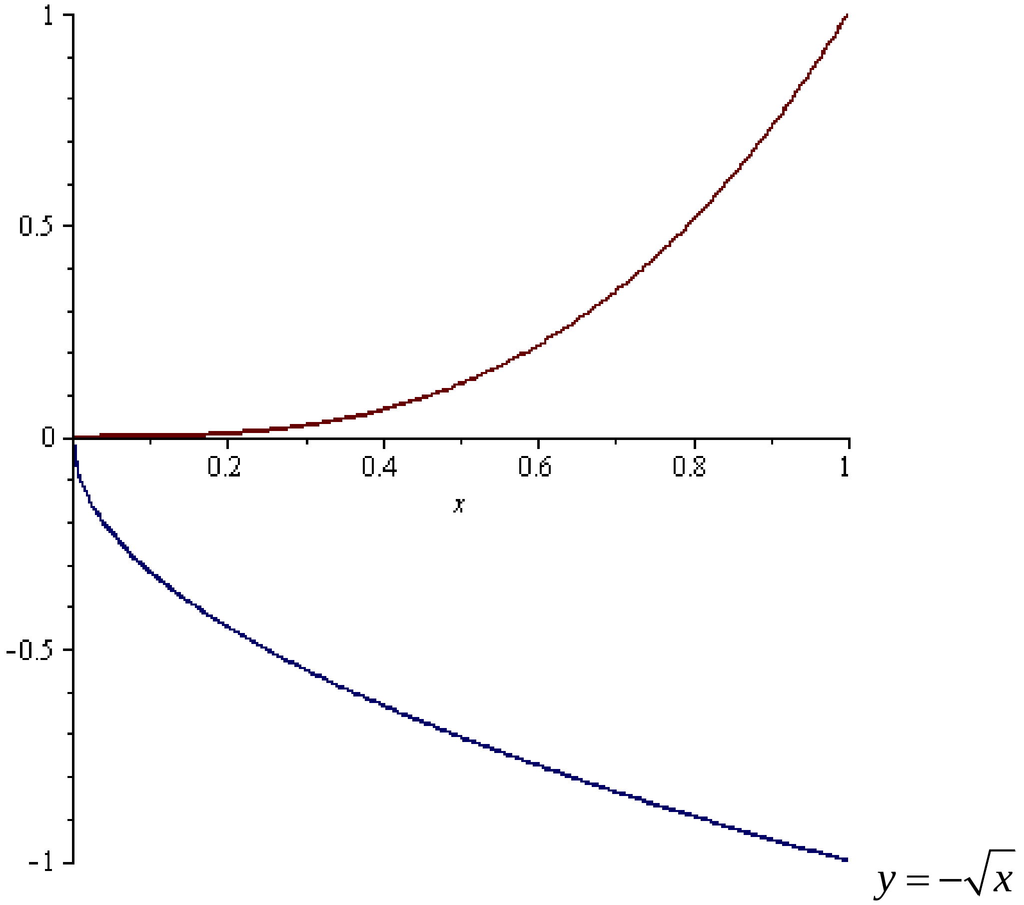
ning qiymati nolga teng bo‘ldi. Shuni endi biz bilgan matematik usul bilan yechib ko‘ramiz.

$$I = \iint_D \left(xy - 4x^3y^3\right) dxdy,\ \ D: x = 1,\ y = x^3,\ y = -\sqrt{x}$$

Birinchi navbatda bizga berilgan D so

hani chizib olamiz.

$$> plot\left(\left[x^3, -x^{\frac{1}{2}}\right], x = 0\,..1\right);\quad y = x^3$$



Demak, D soha quydagicha ekan. $D=\left\{(x,y): 0\le x\le 1,\ -\sqrt{x}\le y\le x^3\right\}$

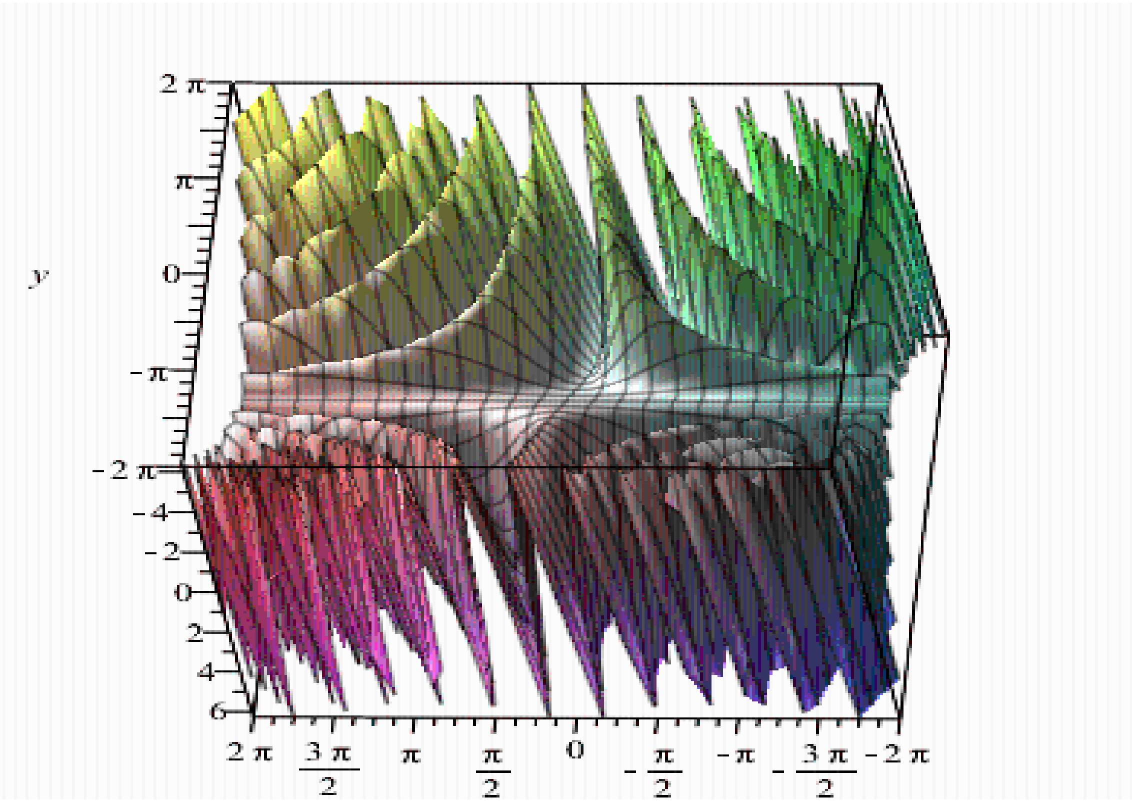
Shundan foydalanib, quydagi ikki karrali integralni tuzamiz va uni hisoblaymiz:

$$I=\iint_D\left(xy-4x^3y^3\right)dxdy=\int_0^1 dx\int_{-\sqrt{x}}^{x^3}\left(xy-4x^3y^3\right)dy=\int_0^1\left(x\cdot\frac{y^2}{2}-x^3y^3\right)\Bigg|_{-\sqrt{x}}^{x^3}dx=$$

$$=\int_0^1\left(\frac{x^7}{2}-x^{15}-\frac{x^2}{2}+x^5\right)dx=\left(\frac{x^8}{16}-\frac{x^{16}}{16}-\frac{x^3}{6}+\frac{x^6}{6}\right)\Bigg|_0^1=0$$

2-misol: $\iint_D y\cos(xy)dxdy,\quad y=\frac{\pi}{2},\ y=\pi,\ x=1,\ x=2$ hisoblang.

> $smartplot3d(y\cdot\cos(x\cdot y))$



Ikki karrali integralni int() buyrug'i yordamida hisoblash.

$$Int\left('(y\cdot\cos(x\cdot y))', y=\frac{\text{pi}}{2}..\text{pi}\right)=int\left(y\cdot\cos(x\cdot y), y=\frac{\text{pi}}{2}..\text{pi}\right);$$

$$\int_{\frac{1}{2}\pi}^{\pi} y\cos(x\,y)\,dy=-\frac{1}{2}\,\frac{\pi\sin\left(\frac{1}{2}x\pi\right)x-2\pi\sin(x\pi)\,x+2\cos\left(\frac{1}{2}x\pi\right)-2\cos(x\pi)}{x^2}$$

$$simplify(rhs(\%));$$

$$-\frac{1}{2}\,\frac{\pi\sin\left(\frac{1}{2}x\pi\right)x-2\pi\sin(x\pi)\,x+2\cos\left(\frac{1}{2}x\pi\right)-2\cos(x\pi)}{x^2}$$

$$Int(\%, x=1..2)=int(\%, x=1..2);$$

$$\int_1^2\left(-\frac{1}{2}\,\frac{\pi\sin\left(\frac{1}{2}x\pi\right)x-2\pi\sin(x\pi)\,x+2\cos\left(\frac{1}{2}x\pi\right)-2\cos(x\pi)}{x^2}\right)dx=-\cos\left(\frac{1}{2}\pi\right)+\frac{3}{2}\cos(\pi)-\frac{1}{2}\cos(2\pi)$$

Ikki karrali integralni Student[MultivariateCalculus] paketining MultiInt() funksiyasi yordamida hisoblash.

with(*Student*[*MultivariateCalculus*]);
$MultiInt\left(y\cdot\cos(x\cdot y), y=\frac{\text{pi}}{2}..\text{pi}, x=1..2, output=steps\right);$

[`.`, *ApproximateInt*, *ApproximateIntTutor*, *AreParallel*, *AreSkew*, *CenterOfMass*, *ChangeOfVariables*, *Contains*, *CrossSection*, *CrossSectionTutor*, *Del*, *DirectionalDerivative*, *DirectionalDerivativeTutor*, *Distance*, *Equal*, *FunctionAverage*, *GetDimension*, *GetDirection*, *GetIntersection*, *GetNormal*, *GetPlot*, *GetPoint*, *GetRepresentation*, *Gradient*, *GradientTutor*, *Intersects*, *Jacobian*, *LagrangeMultipliers*, *Line*, *MultiInt*, *Nabla*, *Plane*, *Revert*, *SecondDerivativeTest*, *SurfaceArea*, *TaylorApproximation*, *TaylorApproximationTutor*]

$$\int_1^2\int_{\frac{\pi}{2}}^{\pi} y\cos(x\,y)\,dy\,dx$$

$$=\int_1^2\left(\left.\frac{\cos(x\,y)+y\sin(x\,y)\,x}{x^2}\right|_{y=\frac{\pi}{2}..\pi}\right)dx$$

$$=\int_1^2 -\frac{\pi\sin\left(\frac{x\,\pi}{2}\right)x-2\,\pi\sin(x\,\pi)\,x+2\cos\left(\frac{x\,\pi}{2}\right)-2\cos(x\,\pi)}{2\,x^2}\,dx$$

$$=1\left(-\frac{\pi\,\mathrm{Si}\left(\frac{x\,\pi}{2}\right)}{2}+\pi\,\mathrm{Si}(x\,\pi)-\frac{\pi\left(-\frac{2\cos\left(\frac{x\,\pi}{2}\right)}{x\,\pi}-\mathrm{Si}\left(\frac{x\,\pi}{2}\right)\right)}{2}+\pi\left(-\frac{\cos(x\,\pi)}{x\,\pi}-\mathrm{Si}(x\,\pi)-\frac{1}{x\,\pi}\right)+\frac{1}{x}\right)\Bigg|_{x=1..2}$$

$$-\cos\left(\frac{1}{2}\pi\right)+\frac{3}{2}\cos(\pi)-\frac{1}{2}\cos(2\,\pi)$$

Ikki karrali integralni Student paketidagi Doubleint() funksiyasi yordamida hisoblash.

$with(student) : Doubleint\left((y\cdot\cos(x\cdot y)), y=\frac{\text{pi}}{2}..\text{pi}, x=1..2\right) : \% = value(\%);$

$$\int_1^2\int_{\frac{1}{2}\pi}^{\pi} y\cos(x\,y)\,dy\,dx=-\cos\left(\frac{1}{2}\pi\right)+\frac{3}{2}\cos(\pi)-\frac{1}{2}\cos(2\,\pi)$$

Ikki karrali integralni VectorCalculus paketining int() funksiyasi yordamida hisoblash.

$$with(VectorCalculus):\ int\left(y\cdot\cos(x\cdot y),[x,y]=Region\left(1..2,\frac{\text{pi}}{2}..\text{pi}\right)\right);$$

$$-\cos\left(\frac{1}{2}\pi\right)+\frac{3}{2}\cos(\pi)-\frac{1}{2}\cos(2\pi)$$

Endi bu misolda ham berilgan D sohani chizamiz.

$$> plot\left(\left[\frac{3.14}{2},3.14\right],x=1..2\right);$$

Demak, bizning D soha quydagicha bo‘ladi.

$$D=\left\{(x,y):1\le x\le 2,\ \frac{\pi}{2}\le y\le\pi\right\}$$

bo‘ladi. Shundan foydalanib quydagicha integral tuzamiz.

$$\iint_D (y\cos(xy))dxdy=\int_{\frac{\pi}{2}}^{\pi}dy\int_1^2(y\cos(xy))dx=\int_{\frac{\pi}{2}}^{\pi}y\frac{1}{y}\sin(xy)_1^2=\int_{\frac{\pi}{2}}^{\pi}(\sin 2x-\sin x)dx=$$

$$=\left(-\frac{1}{2}\cos 2x+\cos x\right)_{\frac{\pi}{2}}^{\pi}=-\frac{1}{2}\cos 2\pi+\cos\pi+\frac{1}{2}\cos\pi-\cos\frac{\pi}{2}=$$

$$=-\frac{1}{2}\cos 2\pi+\frac{3}{2}\cos\pi-\cos\frac{\pi}{2}$$

XULOSA

Demak, xulosa qilsak yuqorida keltirilgan ikki karrali integralni hisoblash bo‘ycha bir nechta usullarni ko‘rib o‘tdik. Bu usullar talabalarga ikki karrali integral ostidagi funksiyani grafigini tassavur qilish imkonyatini taqdim etdi.

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