FURYE ALMASHTIRISHIDAN FOYDALANIB INTEGRALLARNI HISOBLASH

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ANNOTATSIYA

Furye almashtirishlari yordamida matematik analizdagi ba'zi parametrga bog'liq xosmas integrallarni hisoblash muommosi yechiladi. Laplas integrallari va boshqa murakkab integrallar shular jumlasidandir.

Kalit so'zlar*: furye almashtirishlari, Laplas integrallari, absolyut integrallanuvchi, juft funsiya, toq funksiya.*

Furye almashtirishlarining bugungi kunda hayotdagi tatbiqlari juda ko'p bo'lib, biz quyida integrallarni yechishdagi qulayligini qarab chiqamiz.

R sonlar o'qida absolyut integrallanuvchi f funksiya uchun

$$f^*(y) = \int_{-\infty}^{+\infty} f(x)e^{-iyx}\,dx \qquad (1)$$

Integral almashtirish kiritamiz. Bunda f^* funksiya f ning Furye almashtirishi deyiladi.

$$f(x)=\frac{1}{2\pi}\int_{-\infty}^{+\infty} f^*(y)e^{ixy}\,dy \qquad (2)$$

Tenglikning o‘ng tomonidagi birinchi tur xosmas integral yaqinlashsa, u holda f funksiya Furye integraliga yoyiladi va (2) integral teskari Furye almashtirishi deb ham ataladi.

$$e^{-ixy} = \cos xy - i\sin xy$$

Tenlikga ko‘ra,

$$f^*(y) = A(y) - iB(y).$$

$$A(y)=\int_{-\infty}^{+\infty} f(x)\cos xy\,dx \qquad B(y)=\int_{-\infty}^{+\infty} f(x)\sin xy\,dx$$

Yuqoridagilardan foydalanib (2) tenglik quyidagi kurinishni oladi.

$$f(x)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}\big(A(y)-iB(y)\big)(\cos xy + i\sin xy)dy$$

$$=\frac{1}{2\pi}\int_{-\infty}^{+\infty}[A(y)\cos xy + B(y)\sin xy]\,dy.$$

a) Agar $f(x)$ funksiya juft funksiya bo‘lsa, u holda

$$f(x)=\frac{1}{\pi}\int_0^{\infty} A(y)\cos xy\,dy$$

b) Agar $f(x)$ funksiya toq funksiya bo‘lsa, u holda

$$f(x)=\frac{1}{\pi}\int_0^{\infty} B(y)\sin xy\,dy$$

Almashtirishlar o‘rinli bo‘ladi.

1-misol. Laplas inregralini hisoblang. $L_1=\int_0^{\infty}\frac{\cos xy}{1+y^2}dy$.

Yechimi:

$f(x)=\int_0^{\infty}\frac{\cos xy}{1+y^2}dy,$ yuqoridagi a) shartga ko‘ra f(x) funksiya juft funksiya ekanligi kelib chiqadi, a) shart bilan berilgan integralni tenglashtirsak;

$$\frac{1}{\pi}\int_0^\infty A(y)\cos xy\,dy = \int_0^\infty \frac{\cos xy}{1+y^2}dy$$

$$A(y) = \frac{\pi}{1+y^2}\,,\quad A(y) = \int_{-\infty}^{+\infty} f(x)\cos xy\,dx$$

Ekanligi ma’lum, f(x) funksiyani juft funksiya ekanligini inobatga olsak,

$$\frac{\pi}{1+y^2} = 2\int_0^\infty f(x)\cos xy\,dx$$

$$\int_0^\infty f(x)\cos xy\,dx = \frac{\pi}{2}\frac{1}{1+y^2}$$

$\int_0^\infty e^{-ax}\cos bx\,dx = \frac{a}{a^2+b^2}$ ekanligidan foydalanib, f(x) funksiyani juft funksiya ekanligini hisobga olsak, yuqoridagi integral tenglikdan f(x) funksiya quyidagiga tengligi kelib chiqadi;

$f(x) = \frac{\pi}{2}e^{-|x|}$ yani

$$\int_0^\infty \frac{\cos xy}{1+y^2}dy = \frac{\pi}{2}e^{-|x|}.$$

Tenglikka ega bo‘lamiz.

2-misol. Laplas integralini hisoblang. $L_2 = \int_0^\infty \frac{y\sin xy}{1+y^2}dy$

Yechimi: $f(x) = \int_0^\infty \frac{y\sin xy}{1+y^2}dy$

Yuqoridagi b) shartga ko‘ra f(x) funksiya toq funksiya ekanligi kelib chiqadi. B) shart bilan yuqoridagi integrallarni tenglashtirsak,

$$\frac{1}{\pi}\int_0^\infty B(y)\sin xy\,dy = \int_0^\infty \frac{y\sin xy}{1+y^2}dy$$

$$B(y) = \frac{\pi y}{1+y^2},\qquad B(y) = \int_{-\infty}^{+\infty} f(x)\sin xy\,dx$$

Ekanligi ma'lum, f(x) funksiyani toq funksiya ekanligini inobatga olsak;

$$\frac{\pi y}{1+y^2} = 2\int_0^{\infty} f(x)\sin xy\,dx$$

$$\int_0^{\infty} f(x)\sin xy\,dx = \frac{\pi}{2}\frac{y}{1+y^2}$$

$\int_0^{\infty} e^{-ax}\sin bx\,dx = \frac{b}{a^2+b^2}$ tenlikdan va f(x) funksiyani toq funksiya ekanligigan foydalanib yuqoridagi integral tenlikni yechsak, f(x) funksiya quyidagiga teng bo'ladi;

$$f(x) = \frac{\pi}{2} sgnxe^{-|x|}$$

Demak, $\int_0^{\infty}\frac{y\sin xy}{1+y^2}dy = \frac{\pi}{2} sgnxe^{-|x|}$ tenglikka ega b'lamiz.

3-misol. Integralni hisoblang.

$$\int_0^{\infty}\left[\frac{1}{(y-b)^2+a^2}+\frac{1}{(y+b)^2+a^2}\right]\cos xy\,dy$$

Yechimi: $f(x) = \int_0^{\infty}\left[\frac{1}{(y-b)^2+a^2}+\frac{1}{(y+b)^2+a^2}\right]\cos xy\,dy$

Yuqoridagi a) shartga ko'ra f(x) funksiyaning juft funksiya ekanligi kelib chiqadi. A) shartni va yuqoridagi integralni tenglashtirsak,

$$\frac{1}{\pi}\int_0^{\infty} A(y)\cos xy\,dy = \int_0^{\infty}\left[\frac{1}{(y-b)^2\ +a^2}+\frac{1}{(y+b)^2+a^2}\right]\cos xy\,dy$$

$$A(y) = \pi\left[\frac{1}{(y-b)^2+a^2}+\frac{1}{(y+b)^2+a^2}\right], \quad A(y)$$

$$= \int_{-\infty}^{+\infty} f(x)\cos xy\,dx$$

Ekanligi ma'lum, f(x) funksiyani juft funksiya ekanligini inobatga olsak,

$$\int_0^{\infty} f(x)\cos xy\,dx = \frac{\pi}{2}\left[\frac{1}{(y-b)^2+a^2}+\frac{1}{(y+b)^2+a^2}\right]$$

Ushbu tenglikdan f(x) funksiyani topish uchun quyidagi formulalardan foydalanamiz,

$$\cos yx * \cos bx = \frac{1}{2}[\cos(y-b)x + \cos(y+b)x]$$

$$I_1 = \int_0^\infty e^{-ax}\cos(y-b)x\,dx = \frac{a}{(y-b)^2+a^2}$$

$$I_2 = \int_0^\infty e^{-ax}\cos(y+b)x\,dx = \frac{a}{(y+b)^2+a^2}$$

Biz topishimiz kerak bo‘lgan f(x) funksiyaning juftligini va yuqoridagi formulalarni hisobga olsak, f(x) funksiya quyidagiga teng ekanligi kelib chiqadi;

$$f(x) = \frac{\pi}{a}e^{-a|x|}\cos bx$$

Demak, $\int_0^\infty \left[\frac{1}{(y-b)^2+a^2} + \frac{1}{(y+b)^2+a^2}\right]\cos xy\,dy = \frac{\pi}{a}e^{-a|x|}\cos bx$ tenglikka ega bo‘lamiz.

4-misol. Integralni hisoblang.

$$\int_0^\infty \frac{y\sin xy}{[(y-b)^2+a^2][(y+b)^2+a^2]}dy$$

Yechimi: $f(x) = \int_0^\infty \frac{y\sin xy}{[(y-b)^2+a^2][(y+b)^2+a^2]}dy$

Yuqoridagi b) shartga ko‘ra f(x) funksiyaning toq funksiya ekanligi kelib chiqadi. B) shartni va berilgan integrallarni tenglashtirsak,

$$\frac{1}{\pi}\int_0^\infty B(y)\sin xy\,dy = \int_0^\infty \frac{y\sin xy}{[(y-b)^2+a^2][(y+b)^2+a^2]}dy$$

$$B(y) = \frac{\pi y}{[(y-b)^2+a^2][(y+b)^2+a^2]}, \quad B(y) = \int_{-\infty}^{+\infty} f(x)\sin xy\,dx$$

Ekanligi ma’lum, f(x) funksiyaning toq funksiya ekanligini inobatga olsak,

$$\int_0^\infty f(x)\sin xy\,dx = \frac{\pi}{2}\frac{y}{[(y-b)^2+a^2][(y+b)^2+a^2]}$$

$$\int_0^{\infty} f(x)\sin xy\,dx = \frac{\pi}{8b}\left[\frac{1}{(y-b)^2+a^2} - \frac{1}{(y+b)^2+a^2}\right]$$

Ushbu tenglikdan f(x) funksiyani topish uchun quyidagi formulalardan foydalanamiz,

$$\sin yx * \sin bx = \frac{1}{2}[\cos(y-b)x - \cos(y+b)x]$$

Va yuqoridagi I_1, I_2 integrallarni, f(x) funksiyaning toq funksiya ekanligini hisobga olgan holda f(x) funksiyani topsak, u quyidagiga teng bo‘ladi.

$$f(x) = \frac{\pi}{4ab}e^{-a|x|}\sin bx$$

Demak,

$$\int_0^{\infty} \frac{y\sin xy}{[(y-b)^2+a^2][(y+b)^2+a^2]}dy = \frac{\pi}{4ab}e^{-a|x|}\sin bx$$

Tenglikka ega bo‘lamiz

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