VEKTOR FAZOLARDA 2-LOKAL CHIZIQLI OPERATORLAR

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Annotatsiya: Mazkur tezisda vektor fazolar, ularning xususiyatlari, chiziqli operatorlar,2-chiziqli operatorlar va ularning ta'riflari, λ-simmetrik tushunchasi va φ operatorning chiziqliligini isbotlovchi teoremalar bayon etilgan.

Kalit so'zlar: Vektor, vektor fazo, operator, algebra, 2-lokal chiziqli operator, matritsa, λ-simmetrik, endomorfizm.

2-LOCAL LINEAR OPERATORS IN VECTOR SPACES

Abstract: This thesis describes vector spaces, their properties, linear operators, 2-linear operators and their definitions, the concept of λ-symmetry and theorems proving the linearity of the operator φ.

Key words: Vector, vector space, operator, algebra, 2-local linear operator, matrix, λ-symmetric, endomorphism.

Bizga ma'lumki, quyidagi aksiomalarni qanoatlantiradigan V to'plam P maydon ustida chiziqli fazo(vektor fazo) deyiladi [1]:

1) V to'plam qo'shish amaliga nisbatan kommutativ gruppa tashkil etadi;
2) $\forall a \in P, \forall x \in V$ ($ax \in V$ va bir qiymatli);
3) $\forall a \in P, \forall x \in V$ ($ax = xa$);
4) $\forall a, b \in P, \forall x \in V$ ($(ab)x = a(bx)$);
5) $\forall a \in P, \forall x, y \in V$ ($a(x+y) = ax + ay$) va $\forall a, b \in P, \forall x \in V$ ($(a+b)x = ax + bx$);
6) $1 \in P, \forall x \in V$ ($1 \cdot x = x$).

Demak, vektor fazo tushunchasidan foydalanib, chiziqli operator tushunchasini kiritamiz.

P sonli maydon ustida n o‘lchovli V_n chiziqli fazo berilgan bo‘lsin. Faraz qilaylik, V_n ning har bir x vektori biror qoida bo‘yicha shu V_n ning bitta y vektoriga bir qiymatli akslansin. Mana shu qoida algebrada operator deyiladi.

Ta’rif. φ operator quyidagi ikki aksiomaga bo‘ysunsa, uni chiziqli operator deyiladi:

1) $\forall x_1, x_2 \in V_n \ (\varphi(x_1 + x_2) = \varphi x_1 + \varphi x_2)$;

2) $\forall a \in P, \forall x \in V_n \ (\varphi(ax) = a\varphi x)$[2].

Yuqoridagilardan foydalanib, vektor fazolarda 2-lokal chiziqli operatorlar tushunchasiga to‘xtalaylik.

Ta’rif. Ƒ maydon ustidagi V vektor fazo, $\varphi: V \to V$ shunday operator bo‘lsin. V tarkibidagi elementlarning har bir v, w juftligi uchun V ning $\varphi_{v,w}$ endomorfizmi mavjud. Quyidagi shartlar bajarilsa φ 2-lokal chiziqli operator deyiladi:

$\varphi(v) = \varphi_{v,w}(v), \ \varphi(w) = \varphi_{v,w}(w)$.

Ta’rif. V maydon Ƒ maydon ustidagi n o‘lchamli vektor fazo, φ V ning endomorfizmi, $\Lambda = (\lambda_{i,j})_{i,j=1,\ldots,n}$ Ƒ ning nolga teng bo‘lmagan $n \times n$ matritsasi bo‘lsin. V ning bazisi V bo‘lsin. Biz φ ni V ga nisbatan λ-simmetrik deymiz, agar φ ni ifodalovchi $\alpha_v(\varphi)$ matritsa V ga nisbatan quyidagi shaklga ega bo‘lsa:

$\alpha_v(\varphi) = (\lambda_{i,j} a_{i,j})_{i,j=1,\ldots,n}$

bu matritsa

$A = (a_{i,j})_{i,j=1,\ldots,n}$

ga simmetrikdir [3].

Teorema. Ƒ maydon ustidagi n o‘lchamli V vektor fazo, $\varphi : V \to V$ 2-lokal chiziqli operator, $\Lambda = (\lambda_{i,j})_{i,j=1,\ldots,n}$ Ƒ va V ning nolga teng bo‘lmagan $n \times n$ matritsasi, a V ning bazisi. Faraz qilaylik, V dagi har bir v,w uchun V ning $\varphi_{v,w}$ endomorfizmi mavjud, v ga nisbatan Λ-simmetrik va

$\varphi(v) = \varphi_{v,w}(v), \ \varphi(w) = \varphi_{v,w}(w)$

bo‘lsa, φ chiziqlidir [4].

Isbot. $V = (v_1, \ldots, v_n)$ va $X = (x_{i,j})_{i,j=1,\ldots,n}$, quyidagicha aniqlansin:

$\varphi(v_i) = \sum_{j=1}^{n} x_{j,i}v_j \quad i=1,\dots,n$

har bir i,j = 1,…,n uchun $a_{i,j} = x_{i,j} / \lambda_{i,j}$, demak

$\varphi(v_i) = \sum_{j=1}^{n} \lambda_{j,i}\, a_{j,i}\, v_j \quad i=1,\dots,n$

Biz $A=(a_{i,j})_{i,j=1,\dots,n}$ ni simmetrik ekanligini ko‘rsatamiz. $1 \le h < k \le n$ va $\varphi_{vh,vk}$ V ning endomorfizmi, V ga nisbatan Λ-simmetrik bo‘lib, $\varphi(v_h) = \varphi_{vh,vk}(v_h)$, $\varphi(v_k) = \varphi_{vh,vk}(v_k)$. $B = (b_{i,j})_{i,j=1,\dots,n}$ simmetrik matritsa mavjud, shu kabi

$\varphi_{vh,vk}(v_i) = \sum_{j=1}^{n} \lambda_{j,i}\, b_{j,i}\, v_i \quad i=1,\dots,n.$

Bundan kelib chiqadiki,

$\sum_{j=1}^{n} \lambda_{j,h}\, a_{j,h}\, v_j = \varphi(v_h) = \varphi_{vh,vk}(v_h) = \sum_{j=1}^{n} \lambda_{j,h}\, b_{j,h}\, v_j$,

$\sum_{j=1}^{n} \lambda_{j,k}\, a_{j,k}\, v_j = \varphi(v_k) = \varphi_{vh,vk}(v_k) = \sum_{j=1}^{n} \lambda_{j,k}\, b_{j,k}\, v_j$,

bunda

$\sum_{j=1}^{n} \lambda_{j,h}\, a_{j,h}\, v_j = \varphi(v_h) = \varphi_{vh,vk}(v_h) = \sum_{j=1}^{n} \lambda_{j,h}\, b_{j,h}\, v_j$,

$\lambda_{j,k}\, a_{j,k} = \lambda_{j,k}\, b_{j,k} \qquad j = 1,\dots,n.$

Xususan, $\lambda_{k,h}\, a_{k,h} = \lambda_{k,h}\, b_{k,h}$, $\lambda_{h,k}\, a_{h,k} = \lambda_{h,k}\, b_{h,k}$ va $\lambda_{i,j}$ noldan farqli bo‘lgani uchun $a_{k,h} = b_{k,h}$ va $a_{h,k} = b_{h,k}$ bo‘lishi kelib chiqadi. Ammo B simmetrik, shuning uchun $a_{k,h} = b_{k,h} = a_{h,k} = b_{h,k}$ demak, A ham simmetrik.

Endi biz φ ning chiziqli ekanligini ko‘rsatamiz. V da $v= \sum_{i=1}^{n} x_i\, v_i$ va $\varphi(v) = \sum_{i=1}^{n} y_i\, v_i$. φ ning chiziqli ekanligini ko‘rsatishimiz uchun

$y_i = \sum_{j=1}^{n} \lambda_{i,j}\, a_{i,j}\, x_j \quad i=1,\dots,n.$

$1 \le h \le n$ bo‘lsa, V shundayki, V ning Λ-simmetrik bo‘lgan endomorfizmi va $\varphi(v_h) = \varphi_h(v_h)$, $\varphi(v) = \varphi_h(v)$. $B = (b_{i,j})_{i,j=1,\dots,n}$ simmetrik matritsa mavjud. Shu kabi

$\varphi_h(v_i) = \sum_{j=1}^{n} \lambda_{j,i}\, b_{j,i}\, v_j \quad i=1,\dots,n.$

Bundan kelib chiqadiki,

$\sum_{j=1}^{n} \lambda_{j,h}\, a_{j,h}\, v_j = \varphi(v_h) = \varphi_h(v_h) = \sum_{j=1}^{n} \lambda_{j,h}\, b_{j,h}\, v_j$,

shuning uchun

$\lambda_{j,h}\, a_{j,h} = \lambda_{j,h}\, b_{j,h} \qquad j = 1,\dots,n.$

$\lambda_{i,j}$ noldan farqli va A va B simmetrik bo‘lgani uchun biz shunday belgilaymiz:

$a_{h,j} = a_{j,h} = b_{j,h} = b_{h,j} \qquad j = 1,\dots,n.$

φ_h chiziqli bo‘lgani uchun

$\varphi_h(v) = \varphi(v) = \sum_{j=1}^{n} y_i v_i$

dan

$y_i = \sum_{j=1}^{n} \lambda_{i,j} a_{i,j} x_j \quad i=1,\ldots,n$

bo‘lishi kelib chiqadi. Bundan

$y_h = \sum_{j=1}^{n} \lambda_{h,j} b_{h,j} x_j$.

Lekin, $a_{h,j} = b_{h,j}$ uchun $j = 1,\ldots,n$ va bundan

$y_h = \sum_{j=1}^{n} \lambda_{h,j} a_{h,j} x_j$

bo‘ladi va bu teoremani isbotlaydi.

Xulosa o‘rnida shuni aytish mumkinki, $y_h = \sum_{j=1}^{n} \lambda_{h,j} a_{h,j} x_j$ ekanligidan φ ning chiziqli ekanligi kelib chiqdi.

Foydalanilgan adabiyotlar ro‘yxati:

1. https://www.britannica.com/science/vector-space

2. Искандаров Р.И, Назаров Р. АЛГЕБРА ВА СОНЛАР НАЗАРИЯСИ.-Т.:Ўқитувчи, 1977. –Б.245-246

3. Sh. A. Ayupov , F. N. Arzikulov , N. M. Umrzaqov & O. O. Nuriddinov (2020): Description of 2-local derivations and automorphisms on finite-dimensional Jordan algebras, Linear and Multilinear Algebra, DOI: 10.1080/03081087.2020.1845595

4. Cabello J, Peralta A. Weak-2-local symmetric maps on C∗-algebras. Linear Algebra Appl.2016;494:32–43